

Quiz #6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §8.3#17 (5 marks)

Integrate the following indefinite integral:

~~$$\int \cos^2 x \sin^2 x dx$$~~

$$\begin{aligned} &= \int \left(\frac{1 + \cos 2x}{2} \right) \left(\frac{1 - \cos 2x}{2} \right) dx \\ &= \frac{1}{4} \int 1 - \cos^2 2x dx \\ &= \frac{1}{4} \int 1 - \left(\frac{1 + \cos 4x}{2} \right) dx \\ &= \frac{1}{4} \int \frac{1}{2} - \frac{\cos 4x}{2} dx \\ &= \frac{1}{8} \int 1 - \cos 4x dx \end{aligned}$$

using the bonus identities

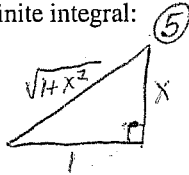
$$\begin{aligned} &= \frac{1}{8} \left[x - \frac{\sin 4x}{4} \right] + C \\ &= \frac{x}{8} - \frac{\sin 4x}{32} + C \end{aligned}$$

Question 2. §8.4#13 (5 marks)

Integrate the following indefinite integral:

$$\int x\sqrt{1+x^2} dx$$

let $x = \tan \theta$ ①
 $dx = \sec^2 \theta d\theta$ ②



① $\int \tan \theta \sqrt{1 + \tan^2 \theta} dx$

② $\int \tan \theta \sqrt{\sec^2 \theta} \sec^2 \theta d\theta$

$= \int \sec \theta \tan \theta \sec^2 \theta d\theta$

let $u = \sec \theta$ ③
 $du = \sec \theta \tan \theta d\theta$ ④

③ $\int u^2 \tan \theta \sec \theta d\theta$

④ $\int u^2 du$

⑤ $= \frac{u^3}{3} + C$
 ③ $= \frac{(\sec \theta)^3}{3} + C$
 ⑤ $= \frac{(\sqrt{1+x^2})^3}{3} + C$

Method 2

let $u = 1+x^2$ ①
 $du = 2x dx$
 $\frac{du}{2} = x dx$ ②

① $\int x \sqrt{u} dx$

② $= \frac{1}{2} \int \sqrt{u} du$

$= \frac{(u)^{3/2}}{3} + C$

$= \frac{(\sqrt{1+x^2})^3}{3} + C$

Bonus. (1 mark)

List the half-angle trigonometric identities given last class.

$$\sin^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$