

## Test 1

This Test is graded out of 60. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (3 marks) Simplify:

$$\frac{(-2xy^{-3})^{-3}}{(3xy^{-1})^{-1}} = \frac{(3xy^{-1})}{(-2xy^{-3})^{+3}} = \frac{3xy^{-1}}{(-2)^{+3}x^{+3}y^{-9}} = \frac{-3xy^4}{8x^3y^4} = -\frac{3y^8}{8x^2}$$

Question 2. (3 marks) Expand and then simplify:

$$\begin{aligned}(2x-3)^3 &= (2x-3)(2x-3)(2x-3) \\ &= (4x^2 - 12x + 9)(2x-3) \\ &= 8x^3 - 24x^2 + 18x - 12x^2 + 36x - 27 \\ &= 8x^3 - 36x^2 + 54x - 27\end{aligned}$$

Question 3. (3 marks) Use long division to find the quotient and remainder:

$$\begin{array}{r} x^3 + 2x^2 - 4 \\ x-3 \end{array} \begin{array}{r} x^2 + 5x + 15 \\ \underline{-(x^3 - 3x^2)} \\ 5x^2 + 0x - 4 \\ \underline{-(5x^2 - 15x)} \\ 15x - 4 \\ \underline{-(15x - 45)} \\ 41 \end{array}$$

$$\frac{x^3 + 2x^2 - 4}{x-3} = x^2 + 5x + 15 + \frac{41}{x-3}$$

Question 4. (1 mark) Factor:

$$x^2 - 16 = (x-4)(x+4)$$

Question 5. (2 marks) Factor:

$$4x^2 - 12x + 9 = (2x - 3)^2$$

Question 6. (1 mark) Factor:

$$x^2 + x - 20 = (x + 5)(x - 4)$$

Question 7. (1 mark) Factor:

$$x^2 + 5x - 36 = (x + 9)(x - 4)$$

Question 8. (3 marks) Factor:

$$\begin{aligned} & 10x^2 - 23x + 12 \\ & = 10x^2 - 8x - 15x + 12 \\ & = 2x(5x - 4) + 3(5x - 4) \\ & = (5x - 4)(2x - 3) \end{aligned}$$

$$10x^2(12) = 120x^2$$

choose  $a, b$  s.t.  $120x^2 = ab$

and  $-23x = a + b$

$$\therefore a = -15x, b = -8x$$

Question 9. (5 marks) Simplify:

$$\begin{aligned} \frac{x^2 - 1}{2x - 4} \times \frac{x^2 - 4}{x^2 - x - 2} \times \frac{3x - 6}{x^2 + x - 2} &= \frac{\cancel{(x-1)}(x+1)}{2\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} \cdot \frac{3\cancel{(x-2)}}{\cancel{(x+2)}(x-1)} \\ &= \frac{3}{2} \end{aligned}$$

Question 10. (2 mark) Solve for x:

$$\begin{aligned}5(x-2) &= 10 - (x+2) \\5x - 10 &= 10 - x - 2 \\6x &= 18 \\x &= 3\end{aligned}$$

Question 11. (2 marks) Rationalize the denominator:

$$\frac{1-\sqrt{3}}{\sqrt{3}} \left( \frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{3}-3}{\sqrt{3}}$$

Question 12. (3 marks) Solve the quadratic equation:

$$\begin{aligned}15x^2 + x - 2 &= 0 \\(3x-1)(5x+2) &= 0 \\x &= \frac{1}{3}, \frac{-2}{5}\end{aligned}$$

Question 13. (3 marks) Solve using the quadratic <sup>formula</sup> equation:

$$\begin{aligned}4x^2 - 20x + 25 &= 0 & x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & &= \frac{20}{8} \\ & & &= \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} & &= \frac{5}{2} \\ & & &= \frac{20 \pm \sqrt{400 - 400}}{8}\end{aligned}$$

Question 14. (3 marks) Solve for x:

$$\begin{aligned}\frac{1}{x+2} - \frac{1}{2} &= \frac{x}{x+2} & \text{LCD: } 2(x+2) \\ \frac{2(x+2)}{2(x+2)} \cdot \frac{1}{x+2} - \frac{2(x+2)}{2(x+2)} \cdot \frac{1}{2} &= \frac{2(x+2)}{2(x+2)} \cdot \frac{x}{x+2} \\ 2 - (x+2) &= 2x \\ 2 - x - 2 &= 2x \\ x &= 0\end{aligned}$$

Question 15. (4 marks) Find the distance and midpoint of the line segment between the points (4, -1) and (-2, -4):

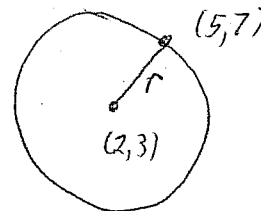
$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2 - 4)^2 + (-4 - (-1))^2} \\
 &= \sqrt{(-6)^2 + (-3)^2} \\
 &= \sqrt{36 + 9} \\
 &= \sqrt{45} \\
 &= \sqrt{9 \cdot 5} \\
 &= 3\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left( \frac{4 - 2}{2}, \frac{-1 - 4}{2} \right) \\
 &= \left( 1, -\frac{5}{2} \right)
 \end{aligned}$$

Question 16. (3 marks) Write the equation of the circle if the center is (2, 3) and (5, 7) is a point on the circumference:

$$\begin{aligned}
 r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(5 - 2)^2 + (7 - 3)^2} \\
 &= \sqrt{3^2 + 4^2} \\
 &= \sqrt{9 + 16} \\
 &= \sqrt{25} \\
 &= 5
 \end{aligned}$$

$$\therefore (x - 2)^2 + (y - 3)^2 = 25 = 5^2$$



Question 17. (4 marks) Find the domain and range of:

$$f(x) = \frac{1}{\sqrt{x-1}} \quad \text{and} \quad \left. \begin{array}{l} x-1 \neq 0 \\ x-1 \geq 0 \end{array} \right\} \Rightarrow \begin{array}{l} x-1 > 0 \\ x > 1 \end{array} \quad \therefore \text{Domain is } (1, \infty)$$

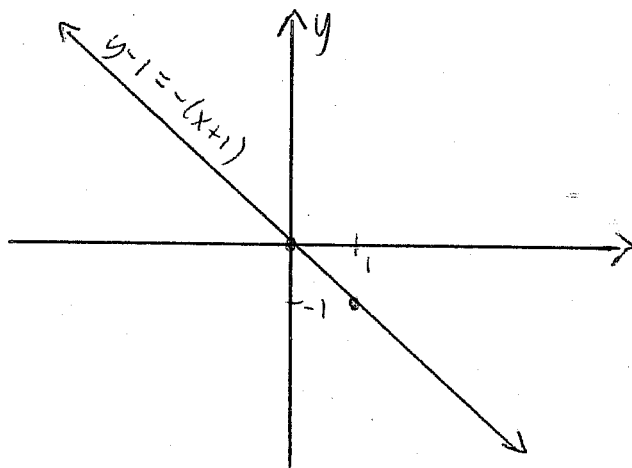
Range is  $(0, \infty)$ .

Question 18. (3 marks) ~~Use the intercepts to graph the following:~~  $y - 1 = -(x + 1)$

Find the intercept and find two points on the line to graph  $y - 1 = -(x + 1)$

Find the intercept, let  $y = 0 \Rightarrow 0 - 1 = -(x - 1) \Rightarrow x = 0$   $\therefore (0, 0)$  is the only axis int.

Find a second point on the line, when  $x = 1 \Rightarrow y - 1 = -(1 + 1)$   
 $y = -2 + 1$   
 $y = -1$   $\therefore (1, -1)$  is a point on the line.



**Question 19.** (4 marks) Find the equation of the line passing through (4, -5) and (8, 1).

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{8 - 4} = \frac{6}{4} = \frac{3}{2}$$

$$y = mx + b$$

$$y = \frac{3}{2}x + b$$

$$1 = \left(\frac{3}{2}\right)(8) + b$$

$$1 = 3(4) + b$$

$$-11 = b$$

$$\therefore y = \frac{3}{2}x - 11$$

**Question 20.** (4 marks) Find the equation of the line passing through (4, -5) and perpendicular to the line  $2y = -(x+1)$ .

Bring  $2y = -(x+1)$  into slope-intercept form  $y = -\frac{1}{2}x + \frac{1}{2}$

Then the slope of the  $\perp$  line is  $-\frac{1}{m} = -1 \Rightarrow m = 2$

$$\therefore y = mx + b$$

$$y = 2x + b$$

$$-5 = 2(4) + b$$

$$-13 = b$$

$$\therefore y = 2x - 13$$

**Bonus.** (3 marks)

Solve for  $x$  in terms of  $k$ :  $k^2x^2 - 3kx - 10 = 0$ .

$$(kx - 5)(kx + 2) = 0$$

$$\therefore x = \frac{5}{k} \text{ and } x = -\frac{2}{k}$$