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Test 1

This Test is graded out of 60. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Simplify:

$$\frac{(-3xy^{-3})^{-3}}{(2xy^{-1})^{-1}} = \frac{(2xy^{-1})}{(-3xy^{-3})^3} = \frac{2xy^{-1}}{-27x^3y^{-9}} = \frac{2\,y^8}{-27\,X^2}$$

Question 2. *(3 marks)* Expand and then simplify:

$$(3x-2)^3 = [9x^2 - 12x + 4](3x-2)$$
$$= 27x^3 - 36x^2 + 12x - 18x^2 + 24x - 8$$
$$= 27x^3 - 36x^2 + 12x$$
$$= 27x^3 - 54x^2 + 36x - 8$$

Question 3. *(3 marks)* Use long division to find the quotient and remainder:

$$\frac{x^3 + 3x^2 - 2}{x - 2}$$

$$
\begin{array}{r}
x^2 + 5x + 10 \\
x-2\overline{\smash{)}\ x^3 + 3x^2 + 0x - 2} \\
-(x^3 - 2x^2) \\
\hline
5x^2 + 0x \\
-(5x^2 - 10x) \\
\hline
10x - 2 \\
-(10x - 20) \\
\hline
18
\end{array}
$$

$$\therefore \quad \frac{x^3 + 3x^2 - 2}{x - 2} = x^2 + 5x + 10 + \frac{18}{x-2}$$

Question 4. *(1 mark)* Factor:

$$x^2 - 9 = (x-3)(x+3)$$

Question 5. *(2 marks)* Factor:

$$9x^2 - 12x + 4 = (3x + 2)^2$$

Question 6. *(1 mark)* Factor:

$$x^2 + x - 30 = (x - 5)(x + 6)$$

Question 7. *(1 mark)* Factor:

$$x^2 + 11x - 26 = (x + 13)(x - 2)$$

Question 8. *(3 marks)* Factor:

$$12x^2 - 23x + 10 = (4x - 5)(3x - 2)$$

$$= 1.2x^2 - 15x - 8x + 10$$
$$= 3x(4x - 5) - 2(4x - 5)$$
$$= (3x - 2)(4x - 5)$$

$12x^2(10) = 120x^2$, choose a, b
s.t. $ab = 120x^2$ and $a + b = -23x$
$\therefore a = -15x$, $b = -8x$

Question 9. *(5 marks)* Simplify:

$$\frac{x^2 - 1}{x^2 - x - 2} \times \frac{3x - 6}{2x - 4} \times \frac{x^2 - 4}{x^2 + x - 2} = \frac{(x + 1)(x - 1)}{(x - 2)(x + 1)} \cdot \frac{3(x - 2)}{2(x - 2)} \cdot \frac{(x - 2)(x + 2)}{(x + 2)(x - 1)}$$

$$= \frac{3}{2}$$

Question 10. *(2 mark)* Solve for x:

$$4(x-1) = 20 - (x+3)$$

$$4x - 4 = 20 - x - 3$$
$$5x = 21$$
$$x = \frac{21}{5}$$

Question 11. *(2 marks)* Rationalize the denominator:

$$\frac{1-\sqrt{5}}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{\sqrt{5} - 5}{5}$$

Question 12. *(3 marks)* Solve the quadratic equation:

$$2x^2 + x - 15 = 0$$

$$(2x - 5)(x + 3) = 0$$

$$\therefore x = -3, \quad x = \frac{5}{2}$$

Question 13. *(3 marks)* Solve using the quadratic ~~equation~~ formula:

$$4x^2 - 20x + 25 = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{20 \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$$

$$= \frac{20 \pm \sqrt{400 - 400}}{8}$$

$$\Rightarrow = \frac{20}{8} = \frac{5}{4}$$

Question 14. *(3 marks)* Solve for *x*:

$$\frac{1}{x+4} - \frac{1}{4} = \frac{x}{x+4} \qquad LCD : 4(x+4)$$

$$4(x+4)\frac{1}{(x+4)} - 4(x+4)\frac{1}{4} = 4(x+4)\frac{x}{(x+4)}$$

$$4 - x - 4 = 4x$$

$$3x = 0$$

$$x = 0$$

$$\therefore x = 0$$

Question 15. *(4 marks)* Find the distance and midpoint of the line segment between the points $(5,-1)$ and $(-3,-4)$:
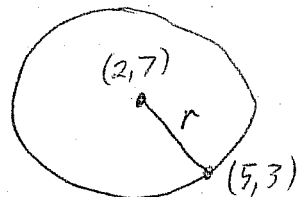
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(-3-5)^2 + (-4-(-1))^2}$$
$$= \sqrt{(-8)^2 + (-3)^2}$$
$$= \sqrt{64 + 9}$$
$$= \sqrt{73}$$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{5-3}{2}, \frac{-1-4}{2}\right)$$
$$= \left(\frac{2}{2}, \frac{-5}{2}\right)$$
$$= \left(1, -\frac{3}{2}\right)$$

Question 16. *(3 marks)* Write the equation of the circle if the center is $(2,7)$ and $(5,3)$ is a point on the circumference:

$$r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(5-2)^2 + (3-7)^2}$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y-7)^2 = 25$$

Question 17. *(4 marks)* Find the domain and range of:

$$f(x) = \frac{1}{\sqrt{2-x}}$$

$$\left.\begin{array}{c} 2-x \neq 0 \\ \text{and} \\ 2-x \geq 0 \end{array}\right\} \Rightarrow \begin{array}{c} 2-x > 0 \\ 2 > x \end{array}$$

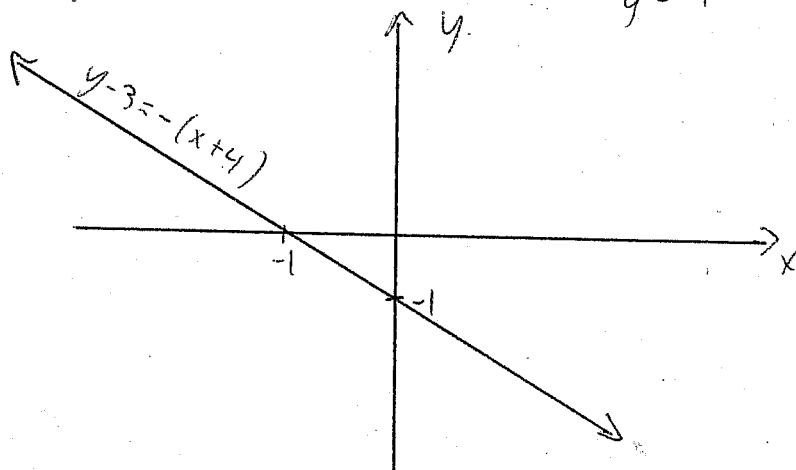$\therefore$ Domain is $(-\infty, 2)$

Range is $(-\infty, 2)$

Question 18. *(3 marks)* Use the intercepts to graph the following: $y - 3 = -(x+4)$

Find the x-intercept, let $y=0$ $\Rightarrow$ $0-3 = -(x+4)$
$$-3 = -x-4$$
$$x = -1$$
$\therefore$ $(-1,0)$ is the x-intercept

Find the y-intercept, let $x=0$ $\Rightarrow$ $y-3 = -(0+4)$
$$y-3 = -4$$
$$y = -1$$
$\therefore$ $(0,-1)$ is the y-intercept

Question 19. *(4 marks)* Find the equation of the line passing through $(4, -6)$ and $(8, 2)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-6)}{8 - 4} = \frac{8}{4} = 2$$

$$\therefore \quad y = mx + b$$
$$y = 2x + b$$

$$y = 2x + b$$
$$2 = 2(8) + b$$
$$2 = 16 + b$$
$$-14 = b$$

$$\therefore \quad y = 2x - 14$$

Question 20. *(4 marks)* Find the equation of the line passing through $(-5, 4)$ and perpendicular to the line $3y = -(x+2)$.

Bring $3y = -(x+2)$ into slope-intercept form $y = \frac{-1}{3}(x+2)$

Then the slope of the $\perp$ line is $\frac{-1}{3}m = -1 \Rightarrow m = 3$

$$\therefore \quad y = mx + b$$
$$y = 3x + b$$
$$4 = 3(-5) + b$$
$$4 + 15 = b$$
$$19 = b$$
$$\therefore \quad y = 3x + 19$$

Bonus. *(3 marks)*
Solve for x in terms of k: $k^2x^2 - 3kx - 10 = 0$.

$$(kx - 5)(kx + 2) = 0$$

$$\therefore \quad x = \frac{5}{k} \quad \text{and} \quad x = \frac{-2}{k}$$