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Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Find the *distance* and the *midpoint* of the line segment between the points $(1,3)$ and $(4,5)$.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(4-1)^2 + (5-3)^2}$$
$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$
$$= \left(\frac{1+4}{2}, \frac{3+5}{2} \right)$$
$$= \left(\frac{5}{2}, \frac{8}{2} \right)$$
$$= \left(\frac{5}{2}, 4 \right)$$

Question 2. *(2 marks)* Find the domain of f.

$$f(x) = \frac{15}{x^3 - 9x} = \frac{15}{x(x^2-9)} = \frac{15}{x(x+3)(x-3)}$$

$$x \neq 0 \qquad x^2 - 9 \neq 0$$
$$x^2 \neq 9$$
$$x \neq \pm 3$$

$$D = (-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$$

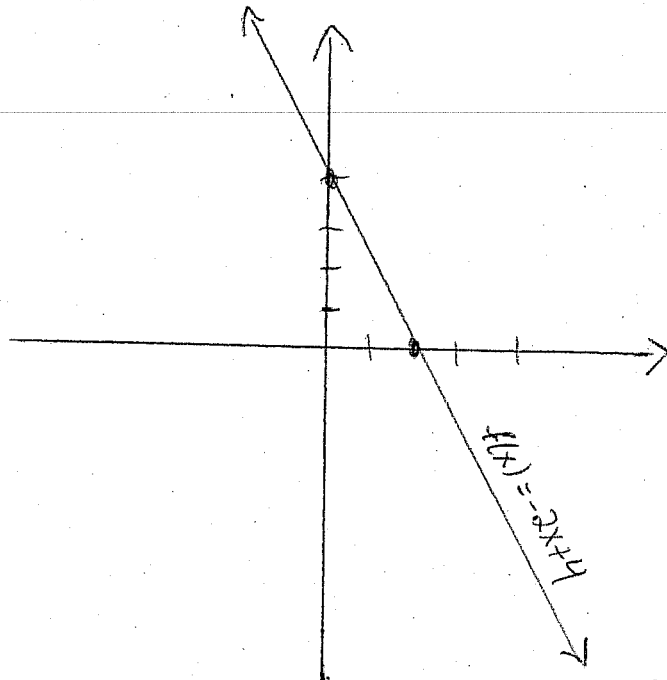**Question 3.** *(4 marks)* Use the x and y intercepts to graph the linear function.

$$f(x) = -2x + 4$$

Let $f(x) = 0 \Rightarrow 0 = -2x + 4$
$$-4 = -2x$$
$$2 = x$$

∴ at $x = 2$ is the x-intercept

Let $x = 0 \Rightarrow f(0) = -2(0) + 4$
$$= +4$$

∴ at $y = +4$ is the y-intercept

Question 4. *(4 marks)* Find the equation of the line that passes through the point $(3,3)$ and is perpendicular to the line $x+4y=10$.

First we rewrite the eqn of the line in y-intercept form $\quad 4y = 10 - x$
$$y = -\frac{1}{4}x + 10$$

$\therefore$ the slope of the line is $-\frac{1}{4}$ and the line perpendicular to it is its negative reciprical $\quad m_1 = \frac{-1}{m} = \frac{-1}{-\frac{1}{4}} = 4$

$\therefore y = m_1 x + b$
$\quad y = 4x + b$, lets determine b by sub $(3,3)$

$3 = 4(3) + b \qquad\qquad \therefore y = 4x - 3$ is $\perp$ to $x + 4y = 10$ and passes
$b = -9 \qquad\qquad\qquad\qquad$ through $(3,3)$

Question 5. *(6 marks)* Graph the parabola $y = -2x^2 + 4x + 3$ and give its intercepts, vertex and range.

Find the vertex of the parabola by completing the square

The y-intercept is at $(0,3)$
$$y = -2x^2 + 4x + 3$$
$$y = -2[x^2 - 2x - \tfrac{3}{2}]$$

To solve for the $\qquad$
we use the quadratic formula.
$$= -2[x^2 - 2x + 1 - 1 - \tfrac{3}{2}]$$
$$= -2[(x-1)^2 - 1 - \tfrac{3}{2}]$$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= -2(x-1)^2 + 2 + 3$$
$$= -2(x-1)^2 + 5 \qquad \therefore \text{vertex} \ (1,5)$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(-2)(3)}}{2(-2)} = \frac{-4 \pm \sqrt{40}}{-4} = \frac{-4 \pm 2\sqrt{10}}{-4} \doteq -0.58 \text{ and } 2.58$$
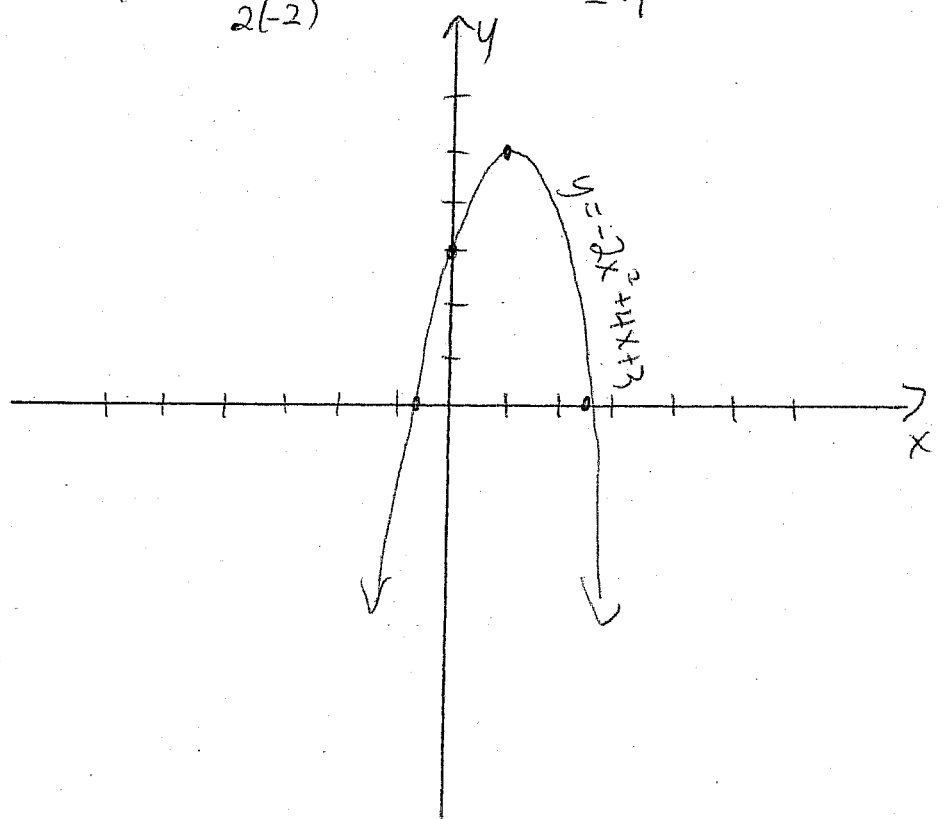
Question 6. *(2 marks)* Find $(f \circ g)(x)$ and $(g \circ f)(x)$ if $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x-1}$.

$$(f \circ g)(x) = f(g(x)) = \frac{1}{\sqrt{x-1}} \qquad (g \circ f)(x) = g(f(x)) = \sqrt{\frac{1}{x} - 1}$$

Question 7. *(4 marks)* Find $f^{-1}(x)$ if $f(x) = 2(x+4)^3 - 1$.

$$y = 2(x+4)^3 - 1$$
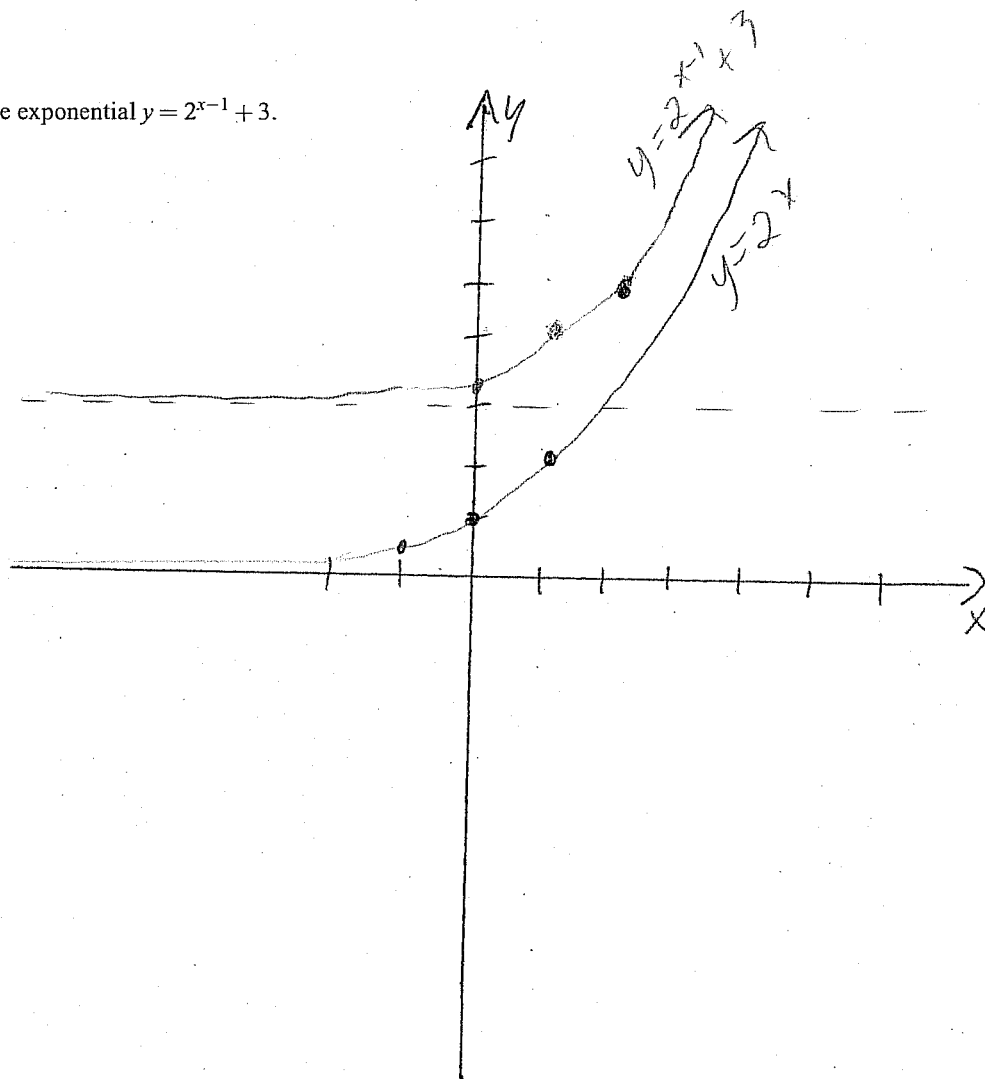$$x = 2(y+4)^3 - 1 \qquad \text{switch } x \text{ and } y$$
$$x + 1 = 2(y+4)^3$$
$$\frac{x+1}{2} = (y+4)^3$$
$$\sqrt[3]{\frac{x+1}{2}} = y + 4$$
$$\sqrt{\frac{x+1}{2}} - 4 = y$$

Question 8. *(4 marks)* Graph the exponential $y = 2^{x-1} + 3$.

x	2^x
-1	$\frac{1}{2}$
0	1
1	2

Question 9. *(5 marks)* Solve for x:

$$\ln(x^2-9) - \ln(x+3) = \ln 2$$

$$\ln \frac{(x^2-9)}{(x+3)} = \ln 2$$

$$\ln \frac{(x-3)(x+3)}{(x+3)} = \ln 2$$

$$\ln(x-3) = \ln 2$$

$$e^{\ln(x-3)} = e^{\ln 2}$$

$$x-3 = 2$$

$$x = 5$$

$$\therefore x = 5$$

Question 10. *(5 marks)* Solve for x:

$$5 + e^{x-3} = 27$$

$$e^{x-3} = 22$$

$$\ln e^{x-3} = \ln 22$$

$$x-3 = \ln 22$$

$$x = \ln 22 + 3$$

$$\therefore x = \ln 22 + 3$$

Bonus. *(3 marks)*
If $f(x) = \frac{x}{x+2}$, find all values of x so that $f(x) = (f \circ f)(x)$.

$$f(x) = f(f(x))$$

$$\frac{x}{x+2} = \frac{\frac{x}{x+2}}{\frac{x}{x+2}+2}$$

$$\frac{x}{x+2} = \frac{\frac{x}{x+2}}{\frac{x+2(x+2)}{x+2}}$$

$$\frac{x}{x+2} = \frac{x \cancel{(x+2)}}{\cancel{x+2}(3x+4)}$$

$$\frac{x}{x+2} = \frac{x}{3x+4}$$

$$x(3x+4) = x(x+2)$$

$$3x^2+4x = x^2+2x$$

$$2x^2+2x = 0$$

$$2x(x+1) = 0$$

$$\therefore x = 0 \text{ and } x = -1$$