Dawson College: **Functions and Trigonometry**: 201-009-50-C1: Winter 2008

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Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Find the *distance* and the *midpoint* of the line segment between the points (2,5) and (1,2).

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1-2)^2+(2-5)^2}$$
$$= \sqrt{1^2 + 3^2}$$
$$= \sqrt{10}$$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{2+1}{2}, \frac{5+2}{2}\right)$$
$$= \left(\frac{3}{2}, \frac{7}{2}\right)$$

Question 2. *(2 marks)* Find the domain of f.

$$f(x) = \frac{11}{x^3-16x} = \frac{11}{x(x^2-16)}$$

$x \neq 0$

$x^2 - 16 \neq 0$
$x^2 \neq 16$
$x \neq \pm 4$

$\therefore D = (-\infty,-4) \cup (-4,0) \cup (0,4) \cup (4,\infty)$

Question 3. *(4 marks)* Use the x and y intercepts to graph the linear function.

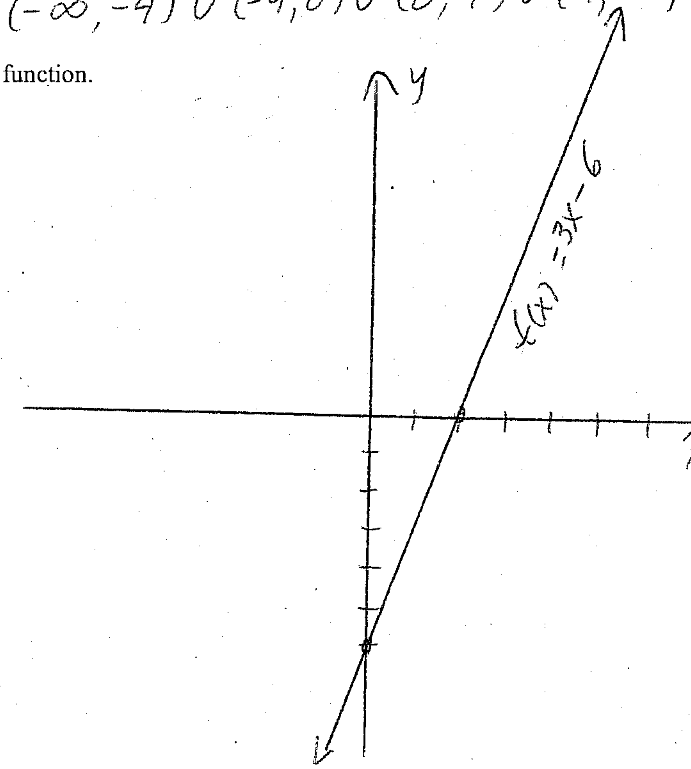
$f(x) = 3x - 6$

Let $f(x) = 0 \Rightarrow 3x - 6 = 0$
$3x = 6$
$x = 2$

$\therefore$ $x = 2$ is the x-intercept

Let $x = 0 \Rightarrow f(0) = 3(0) - 6$
$= -6$

$\therefore$ $y = -6$ is the y-intercept



Question 4. *(4 marks)* Find the equation of the line that passes through the point $(4,5)$ and is perpendicular to the line $x+5y=11$.

First put in y-intercept form $x+5y=11 \iff 5y=-x+11 \iff y=\frac{-x}{5}+\frac{11}{5}$

∴ the slope $m=\frac{-1}{5}$ and the slope of a perpendicular line is the negative reciprical ∴ $m_2=5 \Rightarrow y=5x+b$

We now solve for b.

$$5=5(4)+b$$
$$-16=b$$

∴ $y=5x-16$ is perpendicular to $x+5y=11$ and passes through $(4,5)$

Question 5. *(6 marks)* Graph the parabola $y=-3x^2+6x+2$ and give its intercepts, vertex and range.

To find the vertex by completing the square

$$\begin{aligned} y &= -3x^2+6x+2 \\ &= -3\left[x^2-2x-\tfrac{2}{3}\right] \\ &= -3\left[x^2-2x+\cancel{1}-\cancel{1}-\tfrac{2}{3}\right] \\ &= -3\left[(x-1)^2\right]+3+2 \\ &= -3(x-1)^2+5 \end{aligned}$$

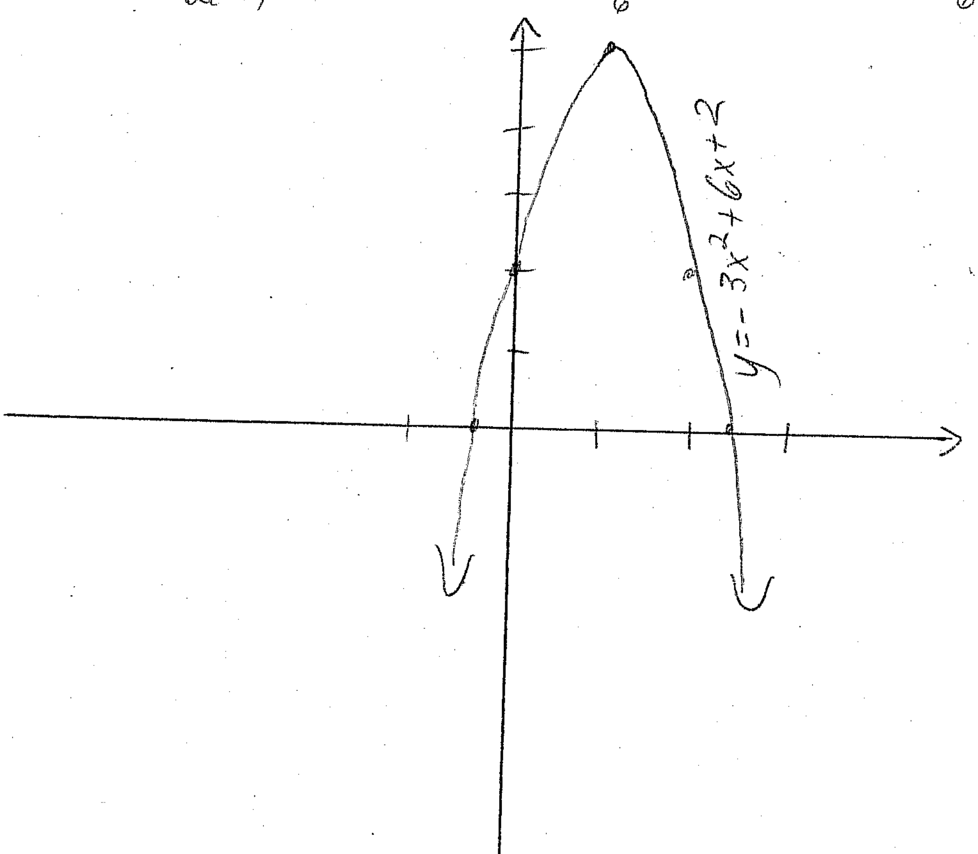
∴ vertex is $(1,5)$

The y-intercept is at $(0,2)$

To solve for the x-intecept we use the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-6\pm\sqrt{36-4(-3)(2)}}{2(-3)}=\frac{-6\pm\sqrt{60}}{-6}=\frac{-6\pm2\sqrt{15}}{-6}\doteq 2{,}29 \text{ and } -0{,}29$$



The range of the parabola is $(-\infty,5]$

Question 6. *(2 marks)* Find $(f \circ g)(x)$ and $(g \circ f)(x)$ if $f(x) = \frac{1}{x-1}$ and $g(x) = \sqrt{x}$.

$(f \circ g)(x) = f(g(x)) = \frac{1}{\sqrt{x}-1}$ $\qquad$ $(g \circ f)(x) = g(f(x)) = \sqrt{\frac{1}{x-1}}$

Question 7. *(4 marks)* Find $f^{-1}(x)$ if $f(x) = 5(x+3)^5 - 2$.

$y = 5(x+3)^5 - 2$ $\qquad$ switch y and x.

$x = 5(y+3)^5 - 2$

$x+2 = 5(y+3)^5$

$\frac{x+2}{5} = (y+3)^5$

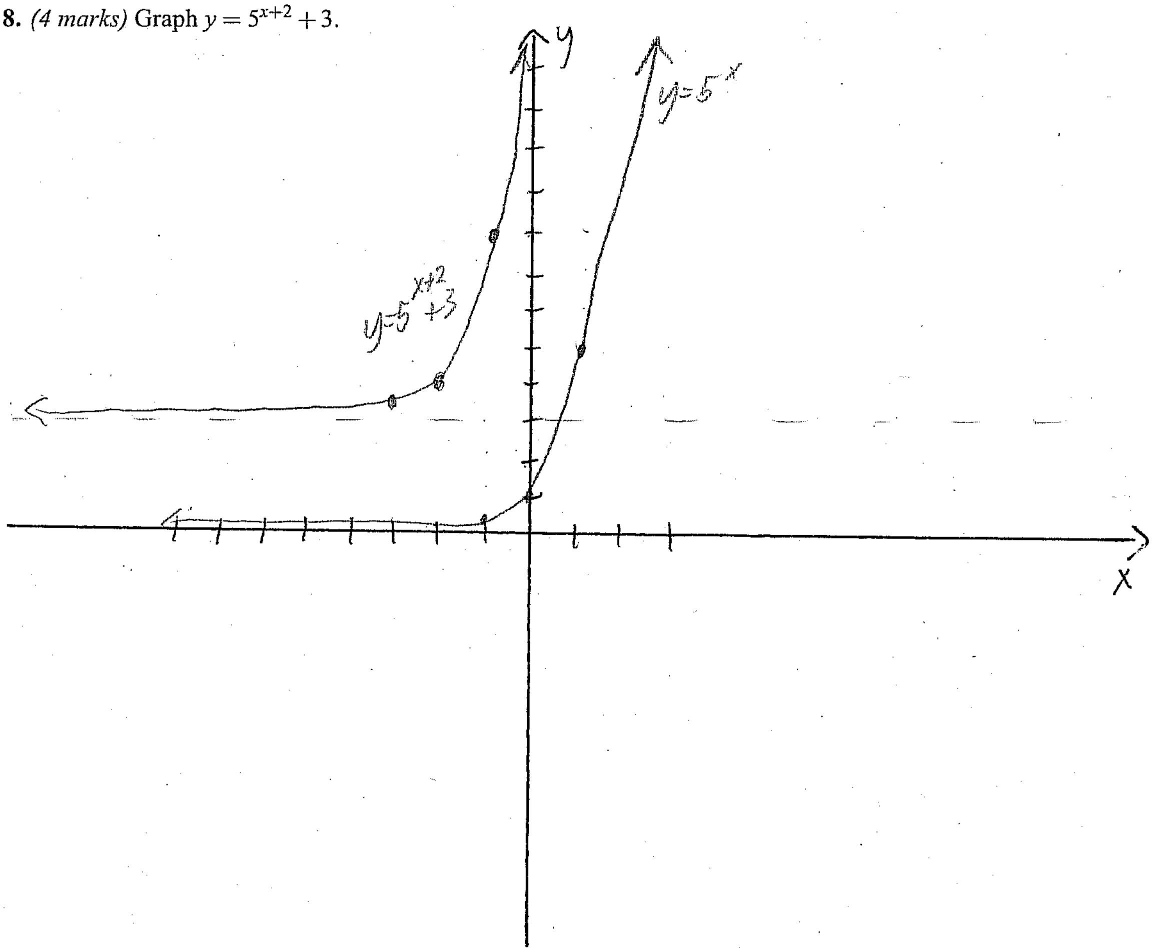
$\sqrt[5]{\frac{x+2}{5}} = y+3$

$\sqrt[5]{\frac{x+2}{5}} - 3 = y$

$f^{-1}(x) = \sqrt[5]{\frac{x+2}{5}} - 3$

Question 8. *(4 marks)* Graph $y = 5^{x+2} + 3$.

x	$y=5^x$
0	1
1	$\frac{1}{5}$
[illegible]	5



Question 9. *(5 marks)* Solve for x:

$\ln(x^2-16)-\ln(x+4)=\ln 2$

$\ln\frac{(x^2-16)}{(x+4)} = \ln 2$

$\ln\frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}} = \ln 2$

$e^{\ln(x-4)} = e^{\ln 2}$

$(x-4) = 2$

$x = 6$

$\therefore x = 6$

Question 10. *(5 marks)* Solve for x:

$2+e^{x-5}=21$

$e^{x-5} = 19$

$\ln e^{x-5} = \ln 19$

$x-5 = \ln 19$

$x = \ln 19 + 5$

Bonus. *(3 marks)*
If $f(x)=\frac{x}{x+2}$, find all values of x so that $f(x)=(f\circ f)(x)$.

$f(x) = f(f(x))$

$\frac{x}{x+2} = \frac{\frac{x}{x+2}}{\frac{x}{x+2}+2}$

$\frac{x}{x+2} = \frac{\frac{x}{x+2}}{\frac{x+2(x+2)}{x+2}}$

$\frac{x}{x+2} = \frac{x\cancel{(x+2)}}{(3x+4)\cancel{(x+2)}}$

$x(3x+4) = x(x+2)$

$3x^2+4x = x^2+2x$

$2x^2+2x = 0$

$2x(x+1) = 0$

$\therefore x=0$ and $x=-1$