

## Test 3

This test is graded out of 44 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** What angle  $\theta$  ( $0^\circ \leq \theta < 360^\circ$ ) is co-terminal to

a. (2 marks)  $1020^\circ$

b. (2 marks)  $\frac{23\pi}{4}$

$$\begin{aligned} a) \quad 1020^\circ &= \theta + K 360^\circ \\ 1020^\circ &= \theta + 2 \cdot 360^\circ \\ 300^\circ &= \theta \end{aligned}$$

$$K = \left\lfloor \frac{1020}{360} \right\rfloor = 2$$

$$b) \quad \frac{23\pi}{4} = \theta + K 2\pi$$

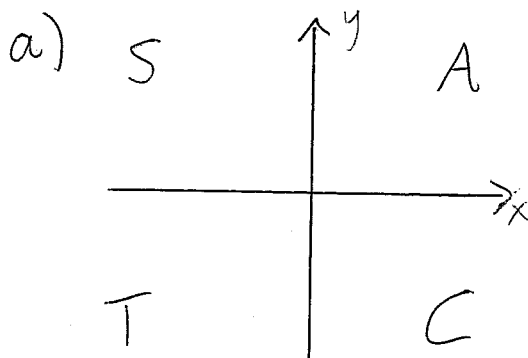
$$K = \left\lfloor \frac{\frac{23\pi}{4}}{2\pi} \right\rfloor = 2$$

$$\begin{aligned} \frac{23\pi}{4} &= \theta + 4\pi \\ \frac{7\pi}{4} &= \theta \end{aligned}$$

**Question 2.** Consider an angle  $\theta$  in standard position, find the quadrant in which its terminal edge lies, if

a. (2 marks)  $\sec \theta < 0$  and  $\cot \theta > 0$

b. (2 marks)  $\sin \theta < 0$  and  $\cos \theta > 0$



Since  $\sec$  is negative it can only be in the second and third quadrant but  $\cot$  is positive therefore it is in the third quadrant

b) Since  $\sin$  is negative the angle can only be in the third and fourth quadrant but  $\cos$  is positive, therefore in the fourth quadrant

**Question 3.** (4 marks) Find the values of the 5 other trigonometric functions, if  $\sin \theta = -\frac{2}{3}$  and  $\sec \theta > 0$ .

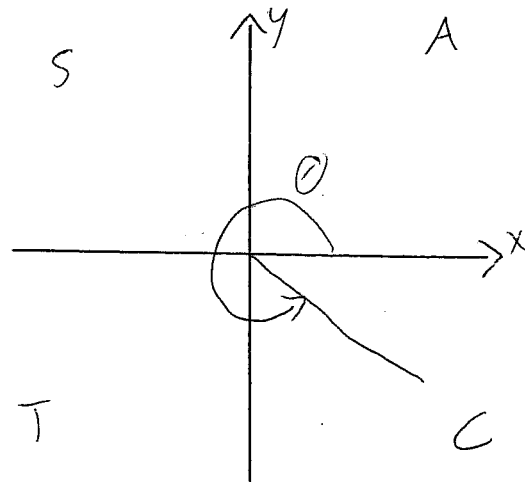
$$\sin \theta = \frac{y}{r} = \frac{-2}{3} \Rightarrow \csc \theta = -\frac{3}{2}$$

$$x = \sqrt{3^2 - 2^2}$$

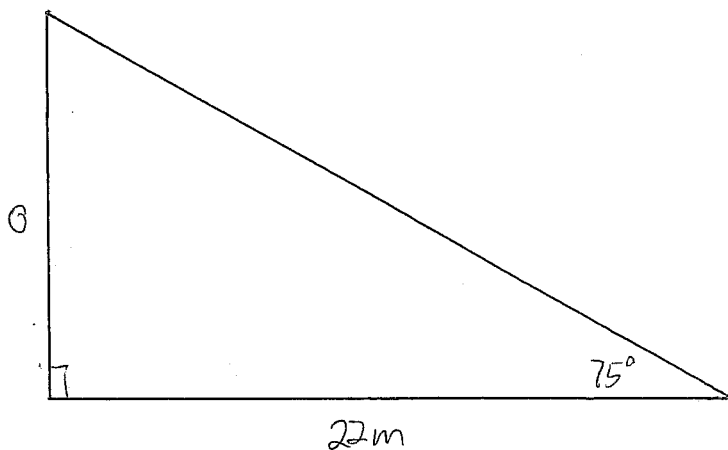
$$x = \sqrt{5}$$

$$\cos \theta = \frac{\sqrt{5}}{3} = \frac{x}{r} \Rightarrow \sec \theta = \frac{3}{\sqrt{5}}$$

$$\tan \theta = \frac{y}{x} = \frac{-2}{\sqrt{5}} \Rightarrow \cot \theta = -\frac{\sqrt{5}}{2}$$



**Question 4.** (4 marks) A forest scientist called Bruno measures the angle of elevation from the ground to the top of a tree and finds the angle to be  $75^\circ$ . If Bruno is 22m from the tree, how tall is the tree?



$$\tan \theta = \frac{o}{a}$$

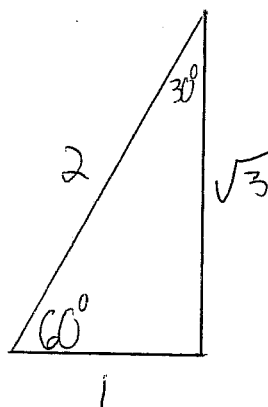
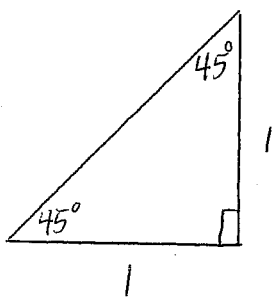
$$\tan 75^\circ = \frac{o}{22}$$

$$22 \tan 75^\circ = o$$

$$82m = o$$

∴ the height of the tree is 82m.

**Question 5.** (4 marks) Draw the two "special triangles" which help identify the special angles. Label the angles of the triangle and the length of each side.

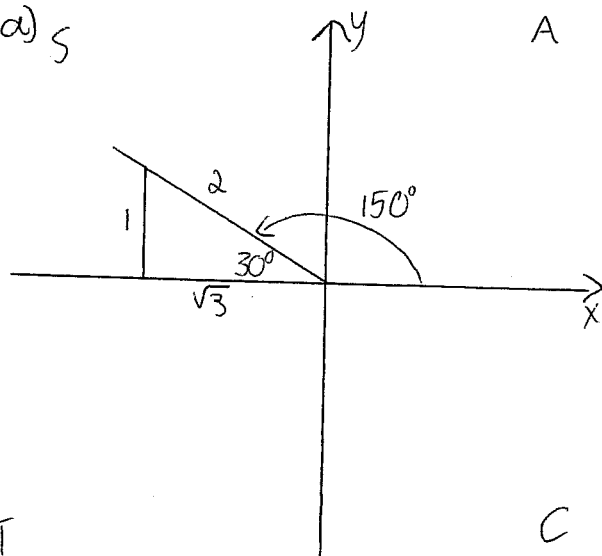


**Question 6.** Find the exact values of

a. (2 marks)  $\sec 150^\circ$

b. (2 marks)  $\tan 300^\circ$

a) 5

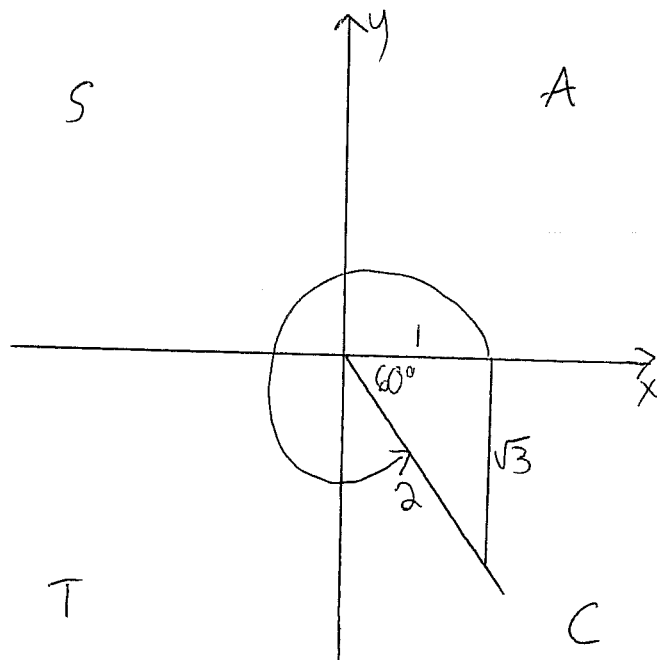


$$\sec 150^\circ = \frac{h}{a} = -\frac{2}{\sqrt{3}}$$

negative since in second quadrant

Soh Cah Toa

b)



$$\tan 300^\circ = \frac{o}{a} = -\sqrt{3}$$

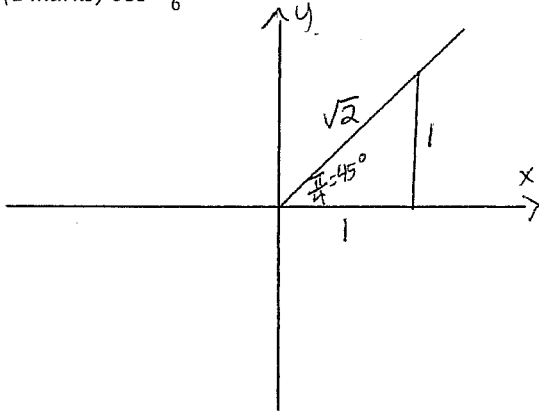
negative since in fourth quadrant.

Question 7. Find the exact values of

a. (2 marks)  $\csc \frac{\pi}{4}$

b. (2 marks)  $\cos \frac{-11\pi}{6}$

a)

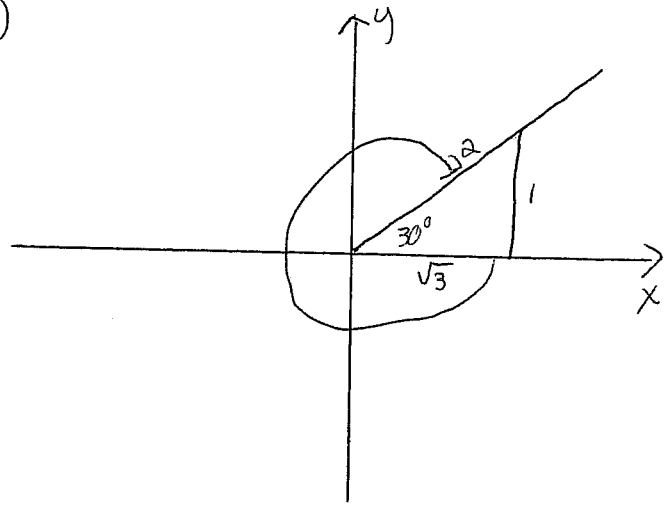


$$\csc \frac{\pi}{4} = \frac{h}{o} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

positive since in first quadrant.

$$\frac{-11\pi}{6} = -330^\circ$$

b)



$$\cos \frac{-11\pi}{6} = \frac{a}{h} = \frac{\sqrt{3}}{2}$$

positive since in first quadrant.

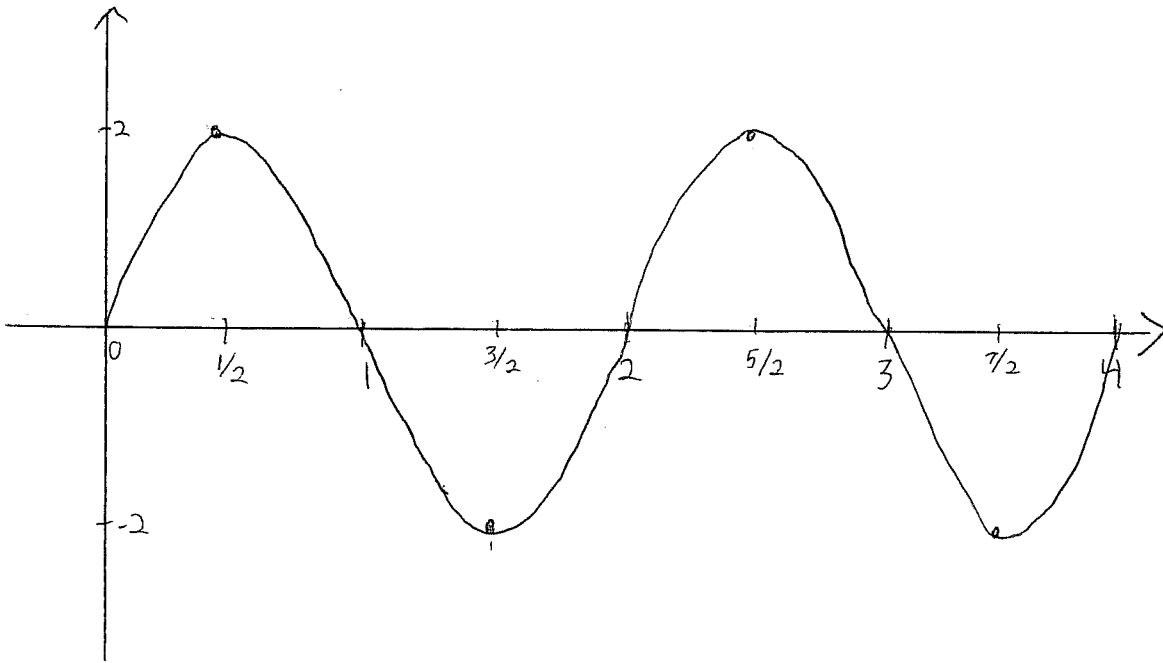
Question 8. (4 marks) Sketch the graph of the function  $f(x) = 2 \sin(\pi x)$  over the interval  $[0, 4]$ .

amplitude =  $|2| = 2$

$$x_i = \frac{i \text{ period}}{4} = \frac{i \cdot 2}{4} = \frac{i}{2}$$

period =  $\frac{2\pi}{\pi} = 2$

$x_0 = 0$	$x_2 = 1$	$x_4 = 2$	$x_6 = 6/2$	$x_8 = 4$
$x_1 = 1/2$	$x_3 = 3/2$	$x_5 = 5/2$	$x_7 = 7/2$	



**Question 9.** (4 marks) Verify the following identity:

$$\tan \theta + \cot \theta \stackrel{?}{=} \sec \theta \csc \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \stackrel{?}{=} \left(\frac{1}{\cos \theta}\right) \left(\frac{1}{\sin \theta}\right)$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \stackrel{?}{=} \frac{1}{\cos \theta \sin \theta}$$

$$\frac{1}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta}$$

Bringing under common denominator  
using pythagorean identity.

**Question 10.** (4 marks) Solve for  $\theta$ :

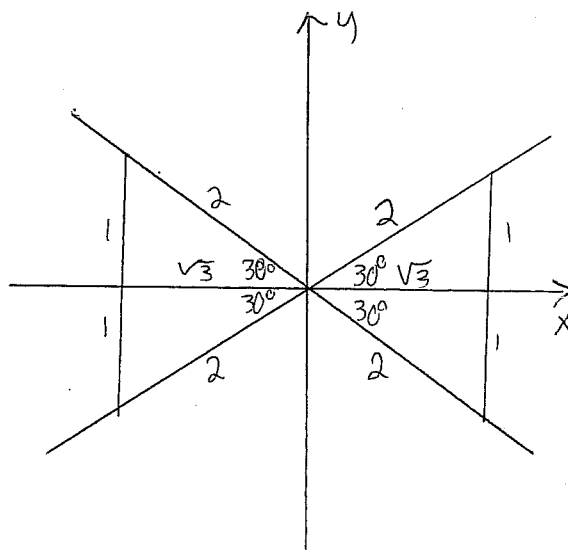
$$4 \cos^2 \theta - 3 = 0$$

$$4 \cos^2 \theta = 3$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$



$$\theta_1 = 30^\circ$$

$$\theta_2 = 180^\circ - 30^\circ = 150^\circ$$

$$\theta_3 = 180^\circ + 30^\circ = 210^\circ$$

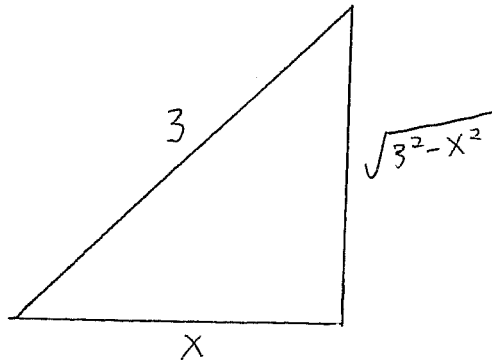
$$\theta_4 = 360^\circ - 30^\circ = 330^\circ$$

Question 11. (4 marks) Find the value of:

$$\tan \left[ \arccos \left( \frac{x}{3} \right) \right]$$

Build a right triangle with

$$\cos \theta = \frac{x}{3} = \frac{a}{h}$$



$$\begin{aligned} \tan \left[ \arccos \left( \frac{x}{3} \right) \right] &= \frac{o}{a} \\ &= \frac{\sqrt{3^2 - x^2}}{x} \end{aligned}$$

Bonus. (2 marks)

Sketch the graph of  $\tan \theta$  over the interval  $[-\pi, \pi]$ .

