

TEST 3

This test is marked out of 65 marks. No books, notes, graphing calculators, or cell phones allowed.
 SHOW ALL YOUR WORK.

Question 1. (4 marks each)

Find the derivative.

- (a) $f(x) = \log_2(x^2)(\sin x)$
- (b) $f(x) = \arctan(\ln x)$
- (c) $f(x) = \cos(3^{4x})$
- (d) $f(x) = \ln(\arcsin x^3)$
- (e) $f(x) = \frac{\arccos x}{4^x}$

(a)

$$\begin{aligned} f'(x) &= \left(\frac{1}{\ln 2} \left(\frac{1}{x^2} \right) 2x \right) \sin x + \cos x (\log_2 x^2) \\ &= \frac{(2 \sin x)}{\ln 2 (x)} + \cos x (\log_2 x^2) \end{aligned}$$

(b) $f'(x) = \frac{1}{\frac{x}{1+(\ln x)^2}} = \frac{1}{x(1+(\ln x)^2)}$

(c) $f'(x) = -\sin(3^{4x}) \cdot (3^{4x} (\ln 3) 4)$

(d) $f'(x) = \frac{1}{\arcsin x^3} \left(\frac{3x^2}{\sqrt{1-x^6}} \right)$

(e) $f'(x) = \frac{-\frac{1}{\sqrt{1-x^2}} (4^x) - (\ln 4) 4^x \arccos x}{(4^x)^2}$

Question 2 (5 marks)

Find the absolute minimum and maximum of the function $g(x) = \sqrt{t^2 + 1}$ on the interval $[-1, 2]$

$$g'(x) = \frac{1}{2\sqrt{t^2+1}} \cdot 2t = \frac{t}{\sqrt{t^2+1}}$$

Critical points $t=0$ is in $[-1, 2]$

$$g(0) = \sqrt{0+1} = 1$$

$$g(-1) = \sqrt{2}$$

$$g(2) = \sqrt{5}$$

so Abs MAX
is $\sqrt{5}$ at $x=2$

& Abs. min is 1
at $x=0$

Question 3. (5 marks)

Use the second derivative test to find and classify the relative extrema of

$$h(t) = 4t^3 - 6t^2 - 24t + 84.$$

$$\begin{aligned} h'(t) &= 12t^2 - 12t - 24 \\ &= 12(t^2 - t - 2) \\ &= 12(t-2)(t+1) \quad \text{critical points} \\ &\quad t=2 \quad t=-1 \end{aligned}$$

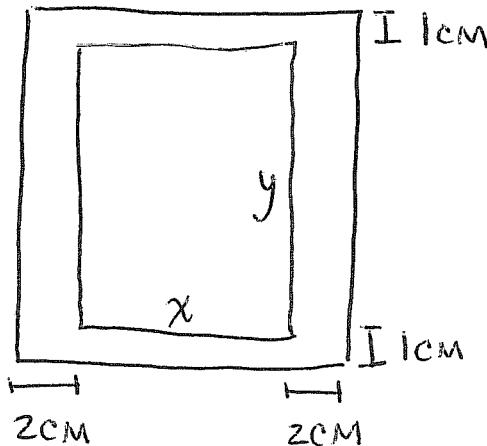
$$h''(t) = 12(2t-1)$$

$$h''(-1) = 12(-3) < 0 \quad \text{so} \quad \begin{array}{l} \text{rel MAX} \\ \text{At } t=-1 \end{array}$$

$$h''(2) = 12(3) > 0 \quad \text{so} \quad \begin{array}{l} \text{rel min.} \\ \text{At } t=2 \end{array}$$

Question 4. (10 marks)

A rectangular page must contain 30cm^2 of printed material. The margins on the top and bottom of the page are 1cm each and the margins on each side of the page are 2cm each. Assuming there can be no print in the margins, and the area of the printed material is a rectangle, find the dimensions of the page with the least amount of area.



$$\begin{aligned}xy &= 30 & y &= 30/x \\A &= (x+4)(y+2) \\A &= (x+4)\left(\frac{30}{x} + 2\right) \\&= 30 + 2x + \frac{120}{x} + 8\end{aligned}$$

$$A' = 2 - \frac{120}{x^2}$$

$$A' = 0 \Rightarrow x^2 = \frac{120}{2}$$

$$\begin{aligned}x^2 &= 60 \\x &= \pm \sqrt{60} \\&= \pm 2\sqrt{15}\end{aligned}$$

$-2\sqrt{15}$

check $x = 2\sqrt{15}$ is a min

INADMISSABLE
(NO NEGATIVE
dimensions)

$A'' = \frac{240}{x^3}$ $A''(2\sqrt{15}) > 0$ so $x = 2\sqrt{15}$ is a min

$$y = \frac{30}{2\sqrt{15}} = \frac{15}{\sqrt{15}}$$

dimensions $x+4 = 2\sqrt{15} + 4 \text{ cm}$

$$8 \quad y+2 = \frac{15}{\sqrt{15}} + 2 \text{ cm}$$

Question 5. (5 marks each)

Compute the following integrals. (a)

$$\int \frac{2x^3 - 2\sqrt{x} + 3}{\sqrt{x}} dx$$

(b)

$$\int (2x^{\frac{3}{2}})(x^3 + x + 2) dx$$

$$\begin{aligned}
 (a) &= \int \frac{2x^3}{x^{\frac{1}{2}}} - \frac{2x^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \frac{3}{x^{\frac{1}{2}}} dx \\
 &= \int 2x^{\frac{5}{2}} - 2 + 3x^{-\frac{1}{2}} dx \\
 &= 2\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) - 2x + 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\
 &= \frac{4}{7}x^{\frac{7}{2}} - 2x + 6x^{\frac{1}{2}} + C
 \end{aligned}$$

$$\begin{aligned}
 (b) &= \int 2x^{\frac{3}{2}}x^3 + 2x^{\frac{3}{2}}x + 2x^{\frac{3}{2}}(2) dx \\
 &= \int 2x^{\frac{9}{2}} + 2x^{\frac{5}{2}} + 4x^{\frac{3}{2}} dx \\
 &= 2\frac{x^{\frac{11}{2}}}{\frac{11}{2}} + 2\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C \\
 &= \frac{4}{11}x^{\frac{11}{2}} + \frac{4}{7}x^{\frac{7}{2}} + \frac{8}{5}x^{\frac{5}{2}} + C
 \end{aligned}$$

Question 6 (15 marks)

Sketch the graph of $f(x) = \frac{2x^2}{x^2 - 4}$.

The first and second derivatives are: $f'(x) = \frac{-16x}{(x^2 - 4)^2}$ and $f''(x) = \frac{16(3x^2 + 4)}{(x^2 - 4)^3}$

Clearly identify on the sketch:

- (a) The x and y intercepts
- (b) Any horizontal and vertical asymptotes
- (c) The intervals where $f(x)$ is increasing/decreasing and any relative extrema
- (d) The intervals where $f(x)$ is concave up/down and any points of inflection

(a) when $x = 0$ $f(x) = \frac{0}{-4} = 0$

when $f(x) = 0$ $x = 0$

one intercept: $(0, 0)$

(b) v.A when denominator = 0
numerator $\neq 0$

v.A at $x = \pm 2$

H.A $\lim_{x \rightarrow \pm\infty} \frac{2x^2}{x^2 - 4} = 2$

H.A at $y = 2$

(c) $f'(x) = 0$ when $x = 0$

doesn't exist when $x = \pm 2$

INTERVAL	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < \infty$
Test	-3	-1	1	3
Sign of f'	+	+	-	-
inc/dec	↗	↗	↘	↘

There is a rel max at $x=0$ $f(x)=0$

(d) $f''(x)$ doesn't exist
At $x=\pm 2$

INTERVALS	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Test	-3	0	3
sign of f''	+	-	+
concavity	U	N	U

NO inflection points because
 f not defined at $x=\pm 2$

