Dawson College: **Calculus II**: 201-203-DW Section 04

Practice Problems 3

Remember to practice using correct notation!

1. Find each limit:

a) $\lim_{x\to\infty} \dfrac{3x^2-2x+5}{5x^2+3x-1}$ **b)** $\lim_{x\to\infty} \dfrac{4x^3-3x^2-5x+1}{2x^3+x^2-5}$ **c)** $\lim_{x\to\infty} \dfrac{4x^2-3x+1}{2x^3-2x^2+5}$

d) $\lim_{x\to\infty} \dfrac{2x^2+x-4}{3x-5}$ **e)** $\lim_{x\to 0} \dfrac{1-e^x}{x^2+1}$ **f)** $\lim_{x\to 0^+} \dfrac{3\sqrt{x}}{\ln(x+1)}$

g) $\lim_{x\to\infty} \dfrac{\ln(3x-10)}{\ln(2x+1)}$ **h)** $\lim_{x\to 1} \dfrac{\ln x}{\sin(\pi x)}$ **i)** $\lim_{x\to 0^+} \dfrac{\sin x}{1-\cos x}$

j) $\lim_{x\to 0} \dfrac{e^x-x-1}{2x^2}$ **k)** $\lim_{x\to\infty} \dfrac{2e^{2x}-2}{e^{3x}-1}$ **l)** $\lim_{x\to 0^+} \dfrac{e^{x-1}-1}{(x-1)^5}$

m) $\lim_{x\to 5} \dfrac{\sin(x^2-25)}{\sin(3x-15)}$

2. Evaluate each improper integral whenever it is convergent.

a) $\int_1^\infty \dfrac{5}{x^4}dx$ **b)** $\int_1^\infty \dfrac{2}{\sqrt{x}}dx$ **c)** $\int_{-\infty}^0 \dfrac{1}{(x-3)^2}dx$

d) $\int_{-\infty}^0 \dfrac{1}{(2-3x)^{3/2}}dx$ **e)** $\int_2^\infty e^{-x/3}$ **f)** $\int_1^\infty \dfrac{x}{x^2-4}dx$

3. Find the area under the curve $y = f(x)$ over the indicated interval.
(a) $f(x) = \frac{1}{(x-2)^2}$,, $x \geq 3$.
(b) $f(x) = \frac{3}{x^{5/3}}$,, $x \geq 5$.
(c) $f(x) = e^{4x}$, $x \leq 2$.

4. Is y is a solution of the differential equation?
(a) $y = e^x - x^2;\ xy' - 2y - (x+2)y''' = 0.$
(b) $y = e^{-2x} - 3xe^{-2x};\ y'' + 2y' - 6y = 18xe^{-2x}.$
(c) $y = \frac{1}{4}e^{2x} + 5;\ 2 - 3y'' + y = 0.$
(d) $y = 2x^2 + 3x - 1;\ 2y' - y'' = 8x + 2.$
(e) $y = e^{x^2};\ y'' - 2xy' - 2y = 0.$

5. Verify that y is the general solution of the differential equation. Find a particular solution that satisfies the side condition.
(a) $y=\frac{C}{x}$; $xy''+y'+\frac{1}{x}y=0$; $y(2)=3$.
(b) $y=Ce^{2x}-2x-1$; $y'-2y-4x=0$; $y(0)=3$

6. Find the general solution of the following first order differential equation.

a) $y'=\dfrac{3x^2-1}{y^2}$ **b)** $y'=\dfrac{2y}{3x+1}$ **c)** $y'=\dfrac{x\cos x^2}{2y+1}$

d) $y'=\dfrac{e^{2x}+2}{\sin y}$ **e)** $y'=\dfrac{y\ln x}{x}$ **f)** $y'=\dfrac{(x-4)y^4}{x^3(y^2-3)}$

7. Find the solution of the initial value problem.

a) $y'=\left(\dfrac{y}{x}\right)^{3/2}$; $y(1)=1$ **b)** $y'=4x^2y-3x^2$; $y(0)=1$

c) $y'=\dfrac{2x^2-3}{4y}$; $y(1)=-2$ **d)** $y'=xe^{-y}$; $y(0)=1$

8. Find the first 3 Taylor polynomials at the indicated number.

a) $f(x)=xe^x$; $x=0$ **b)** $f(x)=\dfrac{1}{1-x}$; $x=2$ **c)** $f(x)=e^x-(x-1)^2$; $x=0$

9. Find the $4th$ Taylor polynomials at the indicated number.

a) $f(x)=e^{-x}$; $x=1$ **b)** $f(x)=\sqrt{1-2x}$; $x=0$

10. Find the first n^{th} Taylor polynomial of $f(x)=\frac{1}{2+x}$ at $x=-1$. Compute $P_4(-1.1)$ and find the error compared with $f(-1.1)$.

11. Determine the convergence of the following sequences. If it converges find the limit.

a) $a_n=\dfrac{n^2+3n-1}{2n^2-n+4}$ **b)** $a_n=\dfrac{2\sqrt[3]{n}-1}{\sqrt[3]{n}+2}$ **c)** $a_n=\left(\dfrac{-1}{4}\right)^n$

d) $a_n=\dfrac{4^n-5}{4^n}$ **e)** $a_n=\dfrac{3^n-5}{4^n}$ **f)** $a_n=\dfrac{3n}{n!}$

g) $a_n=\dfrac{2n+1}{n}-\dfrac{5}{n+6}$ **h)** $a_n=\dfrac{4^n}{6^n}$ **i)** $a_n=\dfrac{2+(-2)^n}{3^n}$

j) $a_n=\dfrac{3^n}{n!}$

12. Determine whether the following series diverge or converge. If it converges find the sum.

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$ **b)** $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)$ **c)** $\displaystyle\sum_{n=0}^{\infty}4\left(-\frac{2}{3}\right)^n$

d) $\sum_{n=0}^{\infty} \frac{(-2)^n}{5^n}$ **e)** $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)} \right)$ **f)** $\sum_{n=0}^{\infty} \frac{3}{2^n}$

g) $\sum_{n=0}^{\infty} \frac{3^n}{5^{n+1}}$ **h)** $\sum_{n=0}^{\infty} 4\left(-\frac{5}{2}\right)^n$ **i)** $3 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

j) $\sum_{n=0}^{\infty} \frac{2+3^n}{5^n}$ **k)** $\sum_{n=0}^{\infty} \frac{3 \cdot 4^n - 2 \cdot 5^n}{6^n}$