

Name: SOLUTIONS

Student ID: _____

Quiz 2

Question 1. (10 marks) Solve the following system of equations:

$$\begin{aligned} 2x_1 + 2x_2 + x_4 &= 4 \\ 2x_1 + 3x_2 + 5x_3 + 4x_4 &= 2 \\ -8x_1 - 10x_2 - 10x_3 - 9x_4 &= -13 \end{aligned}$$

$$\begin{bmatrix} 2 & 2 & 0 & 1 & 4 \\ 2 & 3 & 5 & 4 & 2 \\ -8 & -10 & -10 & -9 & -13 \end{bmatrix} \xrightarrow{R_1 \cdot (\frac{1}{2})} \begin{bmatrix} 1 & 1 & 0 & \frac{1}{2} & 2 \\ 2 & 3 & 5 & 4 & 2 \\ -8 & -10 & -10 & -9 & -13 \end{bmatrix} \xrightarrow{\begin{array}{l} R_2 - 2R_1 \\ R_3 + 8R_1 \end{array}} \begin{bmatrix} 1 & 1 & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 5 & 3 & -2 \\ 0 & -2 & -10 & -5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & \frac{1}{2} & 2 \\ 0 & 1 & 5 & 3 & -2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{\begin{array}{l} R_1 - \frac{1}{2}R_3 \\ R_2 - 3R_3 \end{array}} \begin{bmatrix} 1 & 1 & 0 & 0 & \frac{5}{2} \\ 0 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 0 & -5 & 0 & \frac{3}{2} \\ 0 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

FREE VARIABLES: $x_3 = t$

$$x_4 = -1$$

$$x_2 + 5x_3 = 1$$

$$x_2 = 1 - 5x_3 = 1 - 5t$$

$$x_1 - 5x_3 = \frac{3}{2}, \quad x_1 = \frac{3}{2} + 5x_3 = \frac{3}{2} + 5t$$

SOLUTION SET:

$$(x_1, x_2, x_3, x_4) = \left(\frac{3}{2} + 5t, 1 - 5t, t, -1\right) \quad t \in \mathbb{R}$$