Dawson College: **Linear Algebra**: 201-105-DW Section 05 February 2, 2009

Name: SOLUTIONS
Student ID: _____

Quiz 2

Question 1. *(10 marks)* Solve the following system of equations:

$$\begin{array}{rcrcrcrcl} -x_1 & + & 3x_2 & & & - & x_4 & = & 0 \\ 2x_1 & - & 5x_2 & + & x_3 & - & x_4 & = & 1 \\ & & x_2 & + & x_3 & + & x_4 & = & 1 \\ x_1 & + & x_2 & + & x_3 & & & = & 0 \end{array}$$

AUGMENTED MATRIX:

$$\begin{bmatrix} -1 & 3 & 0 & -1 & 0 \\ 2 & -5 & 1 & -1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \cdot (-1)} \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 2 & -5 & 1 & -1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[R_4 - R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 4 & 1 & -1 & 0 \end{bmatrix}$$

$$\xrightarrow[R_4 - 4R_2]{R_3 - R_2} \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & -3 & 11 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 0 & 0 & -3 & 11 & -4 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \xrightarrow[R_4 \cdot (\frac{1}{4})]{R_2 \cdot (-\frac{1}{3})} \begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 0 & 0 & 1 & -\frac{11}{3} & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow[R_3 + \frac{10}{3}R_4]{\substack{R_1 - R_4 \\ R_2 + 3R_4}} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 - R_3} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 + 3R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{4}{3} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

SOLUTION:

$$(x_1, x_2, x_3, x_4) = (-1, -\tfrac{1}{3}, \tfrac{4}{3}, 0)$$