

## Quiz 5

Question 1. (6 marks) Find the determinant of A.

$$A = \begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & 0 & 2 & 0 \\ 5 & 0 & 5 & -3 \\ 2 & 1 & 4 & 0 \end{bmatrix} \leftarrow \text{row 2}$$

$$\det A = 0 + 0 - 2 \det \begin{bmatrix} 2 & 1 & -1 \\ 5 & 0 & -3 \\ 2 & 1 & 0 \end{bmatrix} + 0$$

$$= -2 \left( -\det \begin{bmatrix} 5 & -3 \\ 2 & 0 \end{bmatrix} + 0 - \det \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \right)$$

$$= -2 \left( - (0 + 6) - (-6 + 5) \right) = -2 (-6 + 1)$$

$$= 10$$

Question 2. (4 marks) Solve the following system using Cramer's Rule.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 4 \\ 3x_1 - x_2 - 2x_3 &= 1 \\ 4x_1 - 3x_3 &= 5 \end{aligned}$$

Given that:

$$\det \begin{bmatrix} 1 & -3 & 1 \\ 3 & -1 & -2 \\ 4 & 0 & -3 \end{bmatrix} = -4, \quad \det \begin{bmatrix} 4 & -3 & 1 \\ 1 & -1 & -2 \\ 5 & 0 & -3 \end{bmatrix} = 38,$$

$$\det \begin{bmatrix} 1 & 4 & 1 \\ 3 & 1 & -2 \\ 4 & 5 & -3 \end{bmatrix} = 22, \quad \det \begin{bmatrix} 1 & -3 & 4 \\ 3 & -1 & 1 \\ 4 & 0 & 5 \end{bmatrix} = 44,$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{38}{-4} = -\frac{19}{2}$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{22}{-4} = -\frac{11}{2}$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{44}{-4} = -11 \quad \therefore (x_1, x_2, x_3) = \left( -\frac{19}{2}, -\frac{11}{2}, -11 \right)$$