

Quiz 6

Question 1. (5 marks) Find the determinant of A by reducing the matrix to row echelon form.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 1 & 5 & 12 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 2 \\ 1 & 5 & 12 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow{R_2 \cdot \left(\frac{1}{2}\right)} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = B \quad \det B = 1$$

$$-\frac{1}{2} \det A = \det B = 1$$

$$\det A = -2$$

Question 2. (2 marks) Find the determinant of the following matrices.

$$A = \begin{bmatrix} 2 & 6 & -1 & 4 \\ -1 & 7 & 0 & -2 \\ 0 & 2 & -3 & 0 \\ 3 & -1 & 8 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\det A = 0$$

$$\det B = -1$$

$$(C_4 = 2C_1)$$

Question 3. (3 marks) Determine the number of inversions in each of the following permutations of $\{1, 2, 3, 4, 5\}$. State whether the permutation is even or odd:

(a) (41352) # OF INVERSIONS = $3 + 0 + 1 + 1 = 5$, ODD

(b) (32541) " " " = $2 + 1 + 2 + 1 = 6$, EVEN

(c) (12345) " " " = 0 EVEN