Name: SOLUTIONS

Student ID: _____

Quiz 8

Question 1. *(2 marks)* Given $\vec{u} = (2, 0, -1)$, $\vec{v} = (-1, -1, 3)$ determine whether the angle between $\vec{u}$ and $\vec{v}$ is acute, obtuse or if they are orthogonal.

$$\vec{u} \cdot \vec{v} = (2, 0, -1) \cdot (-1, -1, 3) = -2 + 0 - 3 = -5 < 0$$

∴ THE ANGLE IS OBTUSE

Question 2. *(5 marks)* Given $\vec{u} = (2, 0, -1)$, $\vec{v} = (-1, -1, 3)$

(a) Find the orthogonal projection of $\vec{u}$ on $\vec{v}$.

$$\vec{w}_1 = \text{proj}_{\vec{v}}\ \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\ \vec{v} = \frac{(2, 0, -1) \cdot (-1, -1, 3)}{(-1, -1, 3) \cdot (-1, -1, 3)} (-1, -1, 3)$$

$$= \frac{-5}{11} (-1, -1, 3) = \left(\frac{5}{11}, \frac{5}{11}, \frac{-15}{11}\right)$$

(b) Decompose $\vec{u}$ into two vectors, one parallel to $\vec{v}$ and one orthogonal to $\vec{v}$. Indicate which vector is parallel and which vector is perpendicular.

$$\vec{w}_2 = \vec{u} - \vec{w}_1 = (2, 0, -1) - \left(\frac{5}{11}, \frac{5}{11}, \frac{-15}{11}\right) = \left(\frac{17}{11}, \frac{-5}{11}, \frac{4}{11}\right)$$

$$\therefore \quad \vec{u} = \vec{w}_1 + \vec{w}_2$$

PARALLEL TO $\vec{v}$ PERPENDICULAR TO $\vec{v}$

Question 2. *(3 marks)* Find the cosine of the angle θ between: $\vec{u} = (1, 2, 0)$, $\vec{v} = (2, -2, 1)$.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{(1, 2, 0) \cdot (2, -2, 1)}{\sqrt{(1)^2 + (2)^2 + (0)^2} \sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$= \frac{-2}{\sqrt{5} \sqrt{9}} = -\frac{2}{3\sqrt{5}}$$