Name: ____SOLUTIONS____

Student ID: _____

Quiz 8

Question 1. *(2 marks)* Given $\vec{u} = (-3, 2, -1)$, $\vec{v} = (-1, 1, 0)$ determine whether the angle between $\vec{u}$ and $\vec{v}$ is acute, obtuse or if they are orthogonal.

$$\vec{u} \cdot \vec{v} = (-3, 2, -1) \cdot (-1, 1, 0) = 3 + 2 + 0 = 5 > 0$$

$$\therefore \text{ THE ANGLE IS ACUTE}$$

Question 2. *(5 marks)* Given $\vec{u} = (-3, 2, -1)$, $\vec{v} = (-1, 1, 0)$

(a) Find the orthogonal projection of $\vec{u}$ on $\vec{v}$.

$$\vec{w_1} = \text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = \frac{(-3, 2, -1) \cdot (-1, 1, 0)}{(-1, 1, 0) \cdot (-1, 1, 0)} (-1, 1, 0)$$

$$= \frac{5}{2} (-1, 1, 0) = \left(-\frac{5}{2}, \frac{5}{2}, 0\right)$$

(b) Decompose $\vec{u}$ into two vectors, one parallel to $\vec{v}$ and one orthogonal to $\vec{v}$. Indicate which vector is parallel and which vector is perpendicular.

$$\vec{w_2} = \vec{u} - \vec{w_1} = (-3, 2, -1) - \left(-\frac{5}{2}, \frac{5}{2}, 0\right)$$

$$= \left(-\frac{1}{2}, -\frac{1}{2}, -1\right)$$

$$\vec{u} = \underset{\text{PARALLEL}}{\vec{w_1}} + \underset{\text{PERPENDICULAR}}{\vec{w_2}}$$

Question 2. *(3 marks)* Find the cosine of the angle θ between: $\vec{u} = (2, 0, -1)$, $\vec{v} = (-1, 1, 1)$.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{(2, 0, -1) \cdot (-1, 1, 1)}{\|(2, 0, -1)\| \|(-1, 1, 1)\|} = \frac{-3}{\sqrt{(2)^2 + 0^2 + (-1)^2} \sqrt{(-1)^2 + 1^2 + 1^2}}$$

$$= \frac{-3}{\sqrt{5}\sqrt{3}} = \frac{-3}{\sqrt{15}}$$