April 15, 2009

Name: SOLUTIONS

Student ID: _____

Quiz 9

Question 1. *(2 marks)* Find a vector normal to both $\vec{u} = (3,2,-1)$ and $\vec{v} = (1,-2,1)$.

$\vec{u} \times \vec{v} = \left(\det\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}, -\det\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}, \det\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}\right)$

$= (0, -4, -8)$

Question 2. *(3 marks)* Find the area of the parallelogram determined by the vectors $\vec{u} = (-2,-1,-1)$ and $\vec{v} = (3,3,4)$.

$\vec{u} \times \vec{v} = \left(\det\begin{bmatrix} -1 & 3 \\ -1 & 4 \end{bmatrix}, -\det\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}, \det\begin{bmatrix} -2 & 3 \\ -1 & 3 \end{bmatrix}\right)$

$= (-1, 5, -3)$

$\text{AREA} = \|\vec{u} \times \vec{v}\| = \sqrt{(-1)^2 + (5)^2 + (-3)^2} = \sqrt{1 + 25 + 9} = \sqrt{35}$ units^2

Question 3. *(4 marks)* Find the volume of the parallelepiped determined by the vectors $\vec{u} = (1,0,-3)$, $\vec{v} = (5,1,1)$, and $\vec{w} = (2,-2,4)$.

$\vec{u} \cdot (\vec{v} \times \vec{w}) = \det\begin{bmatrix} 1 & 0 & -3 \\ 5 & 1 & 1 \\ 2 & -2 & 4 \end{bmatrix}$

$= \det\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} - 3\det\begin{bmatrix} 5 & 1 \\ 2 & -2 \end{bmatrix}$

$= 6 - 3(-12) = 42$

$\text{VOLUME} = |\vec{u} \cdot (\vec{v} \times \vec{w})| = |42| = 42$ units^3