

# Simple Interest

February 24, 2009

## Simple Interest:

- *Interest* is a fee charged for the borrowing of capital (a rent on money). *Simple interest* is governed by the following equation.

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

$$I = Prt$$

where  $I$  is the amount of interest accumulated after  $t$  time (*interest period*, the time unit is measured in years) at the rate  $r$  (percentage per year) for the  $P$  principal sum of money.

- Simple interest is used in short-term loans, savings bonds and purchase on credit.

## Simple Interest: Example

- Compute the interest on the amount of \$2 847.88 at a rate of 7.6% over 122 days.

$$\begin{aligned} I &= Prt \\ &= 2847.88(0.076) \left( \frac{122}{365} \right) \\ &= \$72.34 \end{aligned}$$

- Compute the interest for the amount of \$5 321.23 at a rate of 12.5% over two months.

$$\begin{aligned} I &= Prt \\ &= 5\,321.23(0.125) \left( \frac{2}{12} \right) \\ &= \$110.86 \end{aligned}$$

## Simple Interest: Finding $P$ , $r$ , or $t$

- If the interest  $I$  is given, and two other variables are known one can isolate the unknown variable. Using values of the variables one can determine the unknown value of the variable.

$$I = Prt$$

Isolating the principal:

$$P = \frac{I}{rt}$$

Isolating the rate:

$$r = \frac{I}{Pt}$$

Isolating the time:

$$t = \frac{I}{Pr}$$

## Simple Interest: Finding $P$ , $r$ , or $t$ : Examples

1. What rate of interest will earn \$43.23 if the principal of \$2 040.00 is invested for 129 days?

$$\begin{aligned}I &= Prt \\r &= \frac{I}{Pt} \\r &= \frac{43.23}{2040.00 \left(\frac{129}{365}\right)} \\r &= 6\%\end{aligned}$$

2. What time is required to obtain \$19.34 if the principal of \$1 746.33 is invested at 3.4%?

$$\begin{aligned}I &= Prt \\t &= \frac{I}{Pr} \\t &= \frac{19.34}{1746.33(0.034)} \\t &= 0.3257 \text{ year} \\t &= 119 \text{ days}\end{aligned}$$

## Simple Interest: Finding $P$ , $r$ , or $t$ : Examples

1. What principal amount needs to be invested to earn \$431.23 if the principal is invested for 2 years at a rate of 3.75%?

$$I = Prt$$

$$P = \frac{I}{rt}$$

$$P = \frac{431.23}{0.0375(2)}$$

$$P = \$5\,749.73$$

## Simple Interest: Future Value(Maturity Value)

- *Future value* is the sum of the principal and the interest.

$$\text{Future Value} = \text{Principal} + \text{Interest}$$

$$S = P + I$$

- The above formula can be combined with the equation for simple interest to obtain:

$$S = P + I$$

$$S = P + Prt$$

$$S = P(1 + rt)$$

- **Example:** Emma invested \$10 462.21 for  $5\frac{1}{2}$  years at  $4\frac{1}{4}\%$ . Determine the maturity value of the investment?

$$\begin{aligned} S &= P(1 + rt) \\ &= 10\,462.21(1 + 0.0425(5.5)) \\ &= \$12\,907.75 \end{aligned}$$

## Simple Interest: Present Value

- *Present value* is the required principal needed to obtain a future value. The equation is obtained from the future value equation  $S = P(1 + rt)$  and isolating  $P$ :

$$P = \frac{S}{1 + rt}$$

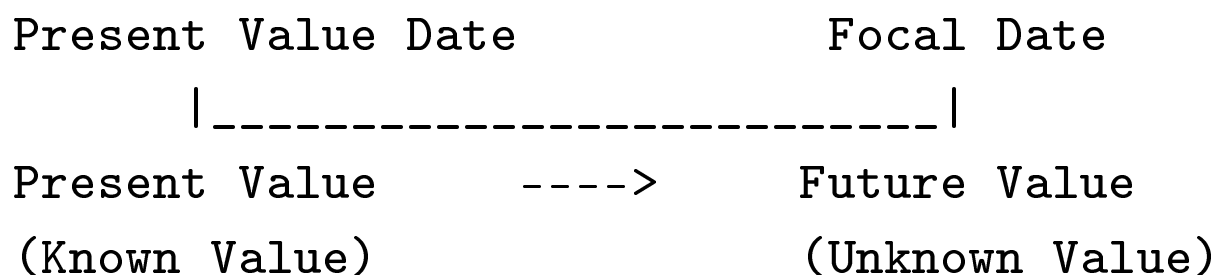
- **Example:** What principal is required to have a future value of \$10 000 in 5 years if the interest rate is 3.25%?

$$\begin{aligned} P &= \frac{S}{1 + rt} \\ &= \frac{10000}{1 + 0.0325(5)} \\ &= \$8\,602.15 \end{aligned}$$



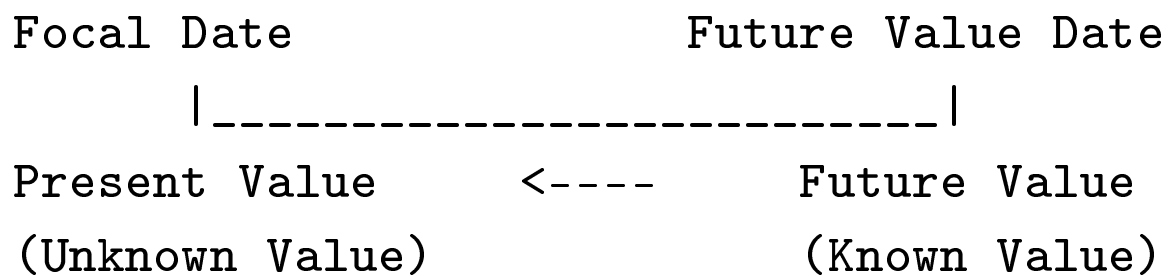
## Simple Interest: Equivalent Values

- Money subject to interest will grow over time. The value of money at a given time is called the *equivalent value*. A method to compare the value of money is to compare its value at a chosen date called the *focal date*.
- If the focal date is after the present value date we use the formula  $S = P(1 + rt)$  to determine the equivalent value at a future time.



It follows that the future value will be greater than the known principal.

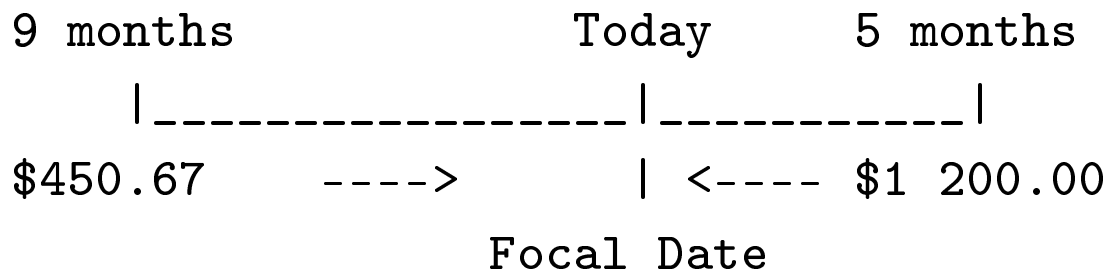
- If the focal date is before the future value date we use the formula  $P = \frac{S}{1+rt}$  to determine the equivalent value at an earlier time.



It follows that the present value will be less than the future value.

## Simple Interest: Equivalent Values: Example: Single Payment

- The Agrarian Bike Shop was owed a payment 9 months ago of \$450.67 at 9.8% and is owed a payment of \$1 200.00 in 5 months at 9.8%. Instead the Agrarian Bike Shop will be given a single equivalent payment today. What is the amount of the single payment?



The equivalent value of \$450.67, today is

$$\begin{aligned}
 S &= P(1 + rt) \\
 &= 450.67 \left( 1 + 0.098 \left( \frac{9}{12} \right) \right) \\
 &= \$483.79
 \end{aligned}$$

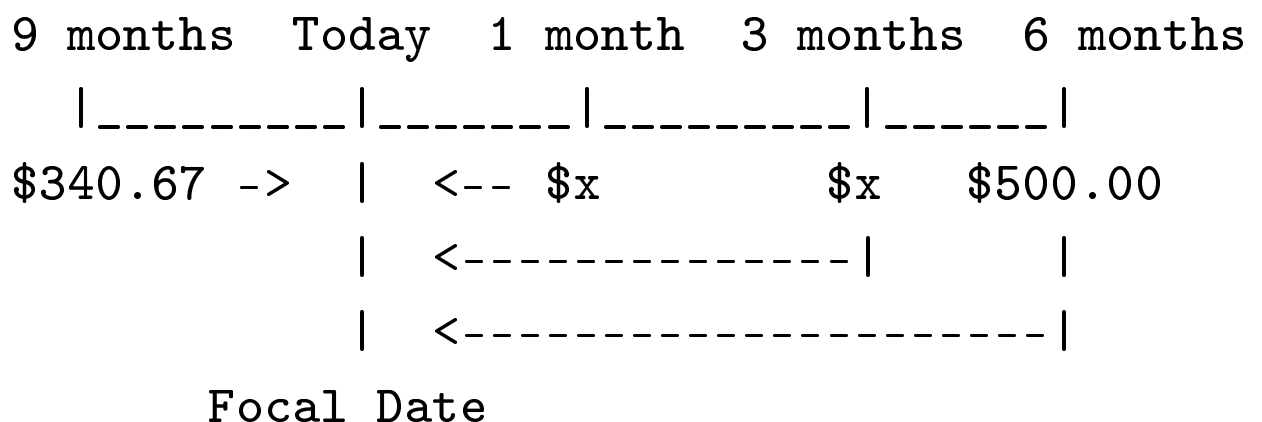
The equivalent value of \$1 200.00, today is

$$\begin{aligned} P &= \frac{S}{1 + rt} \\ &= \frac{1200}{1 + 0.098 \left( \frac{5}{12} \right)} \\ &= \$1\,152.92 \end{aligned}$$

Therefore the amount of the single payment is  $483.79 + 1\,152.92 = \$1\,636.71$

## Simple Interest: Equivalent Values: Example: Equal Payments

- The Agrarian Bike Shop owed a payment 9 months ago of \$340.67 at 12.5% and owes a payment of \$500.00 in 6 months at 11%. Instead the Agrarian Bike Shop will repay the two debts in two equal sized payment. One payment in one month and a second payment in 3 months. Money is now worth 10.5%. What is the size of the equal payments?



where  $x$  is the size of the equal payments. The equiva-

lent value of \$340.67 at 12%, today is

$$\begin{aligned} S &= P(1 + rt) \\ &= 340.67 \left( 1 + 0.125 \left( \frac{9}{12} \right) \right) \\ &= \$372.61 \end{aligned}$$

The equivalent value of \$500.00 at 11%, today is

$$\begin{aligned} P &= \frac{S}{1 + rt} \\ &= \frac{500}{1 + 0.11 \left( \frac{6}{12} \right)} \\ &= \$473.93 \end{aligned}$$

The equivalent values of  $x$  in 1 month and 3 months at 10.5%, respectively, is

$$\begin{aligned} P &= \frac{S}{1 + rt} \\ &= \frac{x}{1 + 0.105 \left( \frac{1}{12} \right)} \\ &= 0.991325898x \end{aligned}$$

$$\begin{aligned}
 P &= \frac{S}{1 + rt} \\
 &= \frac{x}{1 + 0.105 \left( \frac{3}{12} \right)} \\
 &= 0.974421437x
 \end{aligned}$$

To solve for  $x$  we set the replacement payments on one side of the equation and the original payments on the other. We obtain

$$\begin{aligned}
 0.991325898x + 0.974421437x &= 372.61 + 473.93 \\
 1.965747335x &= 846.54 \\
 x &= \$430.65
 \end{aligned}$$

Therefore the size of the equal payments are \$430.65.