Simple Interest

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Simple Interest:

- *Interest* is a fee charged for the borrowing of capital (a rent on money). *Simple interest* is governed by the following quation.

$$\begin{aligned}\text{Interest} &= \text{Principal} \times \text{Rate} \times \text{Time} \\ I &= Prt\end{aligned}$$

  where I is the amount of interest accumulated after t time (*interest period*, the time unit is measured in years) at the rate r (percentage per year) for the P principal sum of money.

- Simple interest is used in short-term loans, savings bonds and purchase on credit.

Simple Interest: Example

- Compute the interest on the amount of $\$2\,847.88$ at a rate of 7.6% over 122 days.

$$
\begin{aligned}
I &= Prt \\
&= 2847.88(0.076)\left(\frac{122}{365}\right) \\
&= \$72.34
\end{aligned}
$$

- Compute the interest for the amount of $\$5\,321.23$ at a rate of 12.5% over two months.

$$
\begin{aligned}
I &= Prt \\
&= 5\,321.23(0.125)\left(\frac{2}{12}\right) \\
&= \$110.86
\end{aligned}
$$

Simple Interest: Finding P, r, or t

- If the interest I is given, and two other variables are known one can isolate the unknown variable. Using values of the variables one can determine the unknown value of the variable.

  $$I = Prt$$

  Isolating the principal:

  $$P = \frac{I}{rt}$$

  Isolating the rate:

  $$r = \frac{I}{Pt}$$

  Isolating the time:

  $$t = \frac{I}{Pr}$$

Simple Interest: Finding P, r, or t: Examples

1. What rate of interest will earn $\$43.23$ if the principal of $\$2\,040.00$ is invested for 129 days?

$$\begin{aligned} I &= Prt \\ r &= \frac{I}{Pt} \\ r &= \frac{43.23}{2040.00\left(\frac{129}{365}\right)} \\ r &= 6\% \end{aligned}$$

2. What time is required to obtain $\$19.34$ if the principal of $\$1\,746.33$ is invested at 3.4%?

$$\begin{aligned} I &= Prt \\ t &= \frac{I}{Pr} \\ t &= \frac{19.34}{1746.33(0.034)} \\ t &= 0.3257 \text{ year} \\ t &= 119 \text{ days} \end{aligned}$$

Simple Interest: Finding P, r, or t: Examples

1. What principal amount needs to be invested to earn $\$431.23$ if the principal is invested for 2 years at a rate of 3.75%?

$$\begin{aligned} I &= Prt \\ P &= \frac{I}{rt} \\ P &= \frac{431.23}{0.0375(2)} \\ P &= \$5\,749.73 \end{aligned}$$

Simple Interest: Future Value(Maturity Value)

- *Future value* is the sum of the principal and the interest.

$$\begin{aligned}\text{Future Value} &= \text{Principal} + \text{Interest} \\ S &= P + I\end{aligned}$$

- The above formula can be combined with the equation for simple interest to obtain:

$$\begin{aligned}S &= P + I \\ S &= P + Prt \\ S &= P(1 + rt)\end{aligned}$$

- **Example:** Emma invested $\$10\,462.21$ for $5\frac{1}{2}$ years at $4\frac{1}{4}\%$. Determine the maturity value of the investment?

$$\begin{aligned}S &= P(1 + rt) \\ &= 10\,462.21(1 + 0.0425(5.5)) \\ &= \$12\,907.75\end{aligned}$$

Simple Interest: Present Value

- *Present value* is the required principal needed to obtain a future value. The equation is obtained from the future value equation $S = P(1+rt)$ and isolating P:

$$P = \frac{S}{1+rt}$$

- **Example:** What principal is required to have a future value of $\$10\,000$ in 5 years if the interest rate is 3.25%?

$$\begin{aligned} P &= \frac{S}{1+rt} \\ &= \frac{10000}{1+0.0325(5)} \\ &= \$8\,602.15 \end{aligned}$$

Simple Interest: Equivalent Values

- Money subject to interest will grow over time. The value of money at a given time is called the *equivalent value*. A method to compare the value of money is to compare its value at a chosen date called the *focal date*.

- If the focal date is after the present value date we use the formula $S = P(1 + rt)$ to determine the equivalent value at a future time.

```
Present Value Date            Focal Date
      |__________________________|
Present Value     ---->     Future Value
(Known Value)               (Unknown Value)
```

It follows that the future value will be greater than the known principal.

- If the focal date is before the future value date we use the formula $P = \frac{S}{1+rt}$ to determine the equivalent value at an earlier time.

```
Focal Date                    Future Value Date
      |__________________________|
Present Value     <----     Future Value
(Unknown Value)             (Known Value)
```

It follows that the present value will be less than the future value.

Simple Interest: Equivalent Values: Example: Single Payment

- The Agrarian Bike Shop was owed a payment 9 months ago of $\$450.67$ at 9.8% and is owed a payment of $\$1\ 200.00$ in 5 months at 9.8%. Instead the Agrarian Bike Shop will be given a single equivalent payment today. What is the amount of the single payment?

```
9 months                Today     5 months
    |_________________|___________|
$450.67    ---->       | <---- $1 200.00
                   Focal Date
```

The equivalent value of $\$450.67$, today is

$$
\begin{aligned}
S &= P(1+rt) \\
&= 450.67\left(1+0.098\left(\frac{9}{12}\right)\right) \\
&= \$483.79
\end{aligned}
$$

The equivalent value of $\$1\ 200.00$, today is

$$\begin{aligned} P &= \frac{S}{1+rt} \\ &= \frac{1200}{1+0.098\left(\frac{5}{12}\right)} \\ &= \$1\ 152.92 \end{aligned}$$

Therefore the amount of the single payment is $483.79 + 1\ 152.92 = \$1\ 636.71$

Simple Interest: Equivalent Values: Example: Equal Payments

- The Agrarian Bike Shop owed a payment 9 months ago of $\$340.67$ at 12.5% and owes a payment of $\$500.00$ in 6 months at 11%. Instead the Agrarian Bike Shop will repay the two debts in two equal sized payment. One payment in one month and a second payment in 3 months. Money is now worth 10.5%. What is the size of the equal payments?

```
9 months  Today  1 month  3 months  6 months
  |_________|_______|_________|______|
$340.67 ->  |  <-- $x        $x   $500.00
            |  <--------------|      |
            |  <---------------------|
       Focal Date
```

where x is the size of the equal payments. The equivalent value of $\$340.67$ at 12%, today is

$$\begin{aligned} S &= P(1+rt) \\ &= 340.67\left(1+0.125\left(\frac{9}{12}\right)\right) \\ &= \$372.61 \end{aligned}$$

The equivalent value of $\$500.00$ at 11%, today is

$$\begin{aligned} P &= \frac{S}{1+rt} \\ &= \frac{500}{1+0.11\left(\frac{6}{12}\right)} \\ &= \$473.93 \end{aligned}$$

The equivalent values of x in 1 month and 3 months at 10.5%, respectively, is

$$\begin{aligned} P &= \frac{S}{1+rt} \\ &= \frac{x}{1+0.105\left(\frac{1}{12}\right)} \\ &= 0.991325898x \end{aligned}$$

$$
\begin{aligned}
P &= \frac{S}{1+rt} \\
&= \frac{x}{1+0.105\left(\frac{3}{12}\right)} \\
&= 0.974421437x
\end{aligned}
$$

To solve for x we set the replacement payments on one side of the equation and the original payments on the other. We obtain

$$
\begin{aligned}
0.991325898x + 0.974421437x &= 372.61 + 473.93 \\
1.965747335x &= 846.54 \\
x &= \$430.65
\end{aligned}
$$

Therefore the size of the equal payments are $\$430.65$.