Dawson College: **Linear Algebra**: 201-105-DW Section 04 February 18, 2009

Last Name: SOLUTIONS

First Name: ______________

Student ID: ______________

Test 1 (A)

Question 1. *(5 marks)* Assume the following matrices in row-echelon form are augmented matrices for systems of equations. Without reducing any further solve the corresponding systems.

a)

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

- $x_3 = -3$
- $x_2 + x_3 = -2 \Rightarrow x_2 = -2 - x_3$
$= -2 + 3 = 1$
- $x_1 + 2x_2 - x_3 = 3$
$x_1 = 3 - 2x_2 + x_3 = 3 - 2(1) + (-3) = -2$

$\therefore (x_1, x_2, x_3) = (-2, 1, -3)$

b)

$$\left[\begin{array}{cccc} 1 & -2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

NO SOLUTION

Question 2. *(10 marks)* Solve the following system of equations:

$$\begin{array}{rcrcrcrcl} & & 3x_2 & & & + & 6x_4 & = & 3 \\ x_1 & + & 3x_2 & - & 3x_3 & + & 2x_4 & = & 1 \\ -2x_1 & - & 3x_2 & + & 6x_3 & + & 2x_4 & = & 1 \end{array}$$

$$\begin{bmatrix} 0 & 3 & 0 & 6 & 3 \\ 1 & 3 & -3 & 2 & 1 \\ -2 & -3 & 6 & 2 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & 3 & -3 & 2 & 1 \\ 0 & 3 & 0 & 6 & 3 \\ -2 & -3 & 6 & 2 & 1 \end{bmatrix} \xrightarrow{R_3 + 2R_1}$$

$$\begin{bmatrix} 1 & 3 & -3 & 2 & 1 \\ 0 & 3 & 0 & 6 & 3 \\ 0 & 3 & 0 & 6 & 3 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 3 & -3 & 2 & 1 \\ 0 & 3 & 0 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \cdot (1/3)} \begin{bmatrix} 1 & 3 & -3 & 2 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 - 3R_2} \begin{bmatrix} 1 & 0 & -3 & -4 & -2 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

FREE VARIABLES:

$x_3 = s, \; x_4 = t$

- $x_1 - 3x_3 - 4x_4 = -2$

  $x_1 = -2 + 3x_3 + 4x_4$

  $= -2 + 3s + 4t$

- $x_2 + 2x_4 = 1$

  $x_2 = 1 - 2x_4$

  $= 1 - 2t$

$\therefore \; (x_1, x_2, x_3, x_4) = (-2+3s+4t, \; 1-2t, \; s, \; t)$

$s, t \in \mathbb{R}$

Question 3. *(10 marks)* Given:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} B = \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix} C = \begin{bmatrix} 0 & 3 \\ -1 & -1 \\ 2 & 2 \end{bmatrix} D = \begin{bmatrix} 2 & 4 \\ 0 & 2 \\ -6 & 2 \end{bmatrix}$$

Computing the following where possible. If not possible indicate so.

a) $5AC$ NOT POSSIBLE

b) $B^T + A$

$$= \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix}^T + \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

c) $CB - \frac{1}{2}D$

$$= \begin{bmatrix} 0 & 3 \\ -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 0 & 2 \\ -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 3 \\ 1 & -4 \\ -2 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 1 & -5 \\ 1 & 7 \end{bmatrix}$$

Question 4. *(10 marks)* Find A given:

$$(7A-I)^{-1} = \begin{bmatrix} 5 & 1 \\ 6 & 1 \end{bmatrix}$$

$$7A - I = [(7A-I)^{-1}]^{-1} = \begin{bmatrix} 5 & 1 \\ 6 & 1 \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 6 & -5 \end{bmatrix}$$

$$7A = \begin{bmatrix} -1 & 1 \\ 6 & -5 \end{bmatrix} + I = \begin{bmatrix} -1 & 1 \\ 6 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 6 & -4 \end{bmatrix}$$

$$A = \frac{1}{7}\begin{bmatrix} 0 & 1 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1/7 \\ 6/7 & -4/7 \end{bmatrix}$$

Question 5. *(5 marks)* Given $n \times n$ invertable matrices A B, and C and scalar k state whether the following statements are always true (T) or not (F).

a) $(A^T)^T = A$ T
b) $(AB)^{-1} = B^{-1}A^{-1}$ T
c) $AB = BA$ F
d) $(kA)^T = kA^T$ T
e) $(A+B)^{-1} = A^{-1} + B^{-1}$ F

Question 6. *(10 marks)* Given:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & -4 \\ 5 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & -2 \\ 5 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & -4 \\ 3 & -2 & -5 \end{bmatrix}$$

Find Elementary matrices E_1, E_2, E_3 such that:

a) $E_1 A = B$.

$$A \xrightarrow{R_2 \cdot (\frac{1}{2})} B \quad \Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \cdot (\frac{1}{2})} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $E_2 A = C$.

$$A \xrightarrow{R_3 - 2R_1} C \quad \Rightarrow \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

c) $E_3 C = A$. Without multiplying state the result of $E_2 E_3$.

$$C \xrightarrow{R_3 + 2R_1} A \quad \Rightarrow \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$E_2 E_3 = I$ SINCE E_2 AND E_3 WERE OBTAINED FROM INVERSE ELEMENTARY ROW OPERATIONS $\Rightarrow$ E_3 AND E_2 ARE INVERSES

Question 7. *(10 marks)*
Find A^{-1} (if possible) given:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 3 & -2 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+R_1]{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & -1 & 1 \end{array}\right] \xrightarrow{R_2+2R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 4 & -1 & 2 \\ 0 & 0 & 1 & 3 & -1 & 1 \end{array}\right] \xrightarrow{R_1-R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 1 & -2 \\ 0 & 1 & 0 & 4 & -1 & 2 \\ 0 & 0 & 1 & 3 & -1 & 1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} -3 & 1 & -2 \\ 4 & -1 & 2 \\ 3 & -1 & 1 \end{bmatrix}$$

Bonus: *(1 marks)* Recommend an interesting item that you would not leave without when going on a long backpacking trip. Remember, space in your bag is an issue. If time permits, explain your answer.