Dawson College: **Linear Algebra**: 201-105-DW Section 05 February 16, 2009

Last Name: SOLUTIONS

First Name:

Student ID:

Test 1 (A)

Question 1. *(10 marks)*

a) Briefly state the three types of elementary row operations.

1) INTERCHANGE 2 ROWS

2) MULTIPLY A ROW BY A NONZERO CONSTANT

3) ADD A MULTIPLE OF A ROW TO ANOTHER ROW

b) Using elementary row operations, find the reduced row-echelon form of the following matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 1 & -5 & -1 \\ 2 & 2 & -6 & 6 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & 1 & -5 & -1 \\ 0 & 0 & 1 & 2 \\ 2 & 2 & -6 & 6 \end{bmatrix} \xrightarrow{R_2 - 2R_1}$$

$$\begin{bmatrix} 1 & 1 & -5 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 8 \end{bmatrix} \xrightarrow{R_3 - 4R_2} \begin{bmatrix} 1 & 1 & -5 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + 5R_2}$$

$$\begin{bmatrix} 1 & 1 & 0 & 9 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2. *(10 marks)* The following is an augmented matrix for a system of equations in row echelon form.

$$\begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 6 & 9 \\ 0 & 0 & a-6 & b+7 \end{bmatrix}$$

What are the values of a and b so that the system has:
a) no solutions?

$a-6=0$ AND $b+7=1$

$\boxed{a=6 \quad \text{AND} \quad b=-6}$

b) infinitely many solutions?

$a-6=0$ AND $b+7=0$

$\boxed{a=6 \quad \text{AND} \quad b=-7}$

c) one solution?

$a-6=1$ AND b ANY NUMBER

$\boxed{a=7}$

Question 3. *(10 marks)* Solve the following system of equations:

$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 - 4x_4 &= -2 \\ x_2 + 3x_3 + 3x_4 &= 1 \\ 2x_1 + 4x_2 + 8x_3 + 2x_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 2 & 2 & 2 & -4 & -2 \\ 0 & 1 & 3 & 3 & 1 \\ 2 & 4 & 8 & 2 & 0 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 2 & 2 & 2 & -4 & -2 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 2 & 6 & 6 & 2 \end{bmatrix} \xrightarrow{R_3 - 2R_2}$$

$$\begin{bmatrix} 2 & 2 & 2 & -4 & -2 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \cdot (1/2)} \begin{bmatrix} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - R_2}$$

$$\begin{bmatrix} 1 & 0 & -2 & -5 & -2 \\ 0 & 1 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

FREE VARIABLES:

$x_3 = s \; , \; x_4 = t$

- $x_1 - 2x_3 - 5x_4 = -2$

$x_1 = -2 + 2x_3 + 5x_4$

$= -2 + 2s + 5t$

- $x_2 + 3x_3 + 3x_4 = 1$

$x_2 = 1 - 3x_3 - 3x_4$

$= 1 - 3s - 3t$

SOLUTION SET:

$(x_1, x_2, x_3, x_4) = (-2 + 2s + 5t, \; 1 - 3s - 3t, \; s, \; t)$

$s, t \in \mathbb{R}$

Question 4. *(10 marks)* Given:

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 \\ -1 & -1 \end{bmatrix}$$

Computing the following where possible. If not possible indicate so.

a) $A^T + B$

$$= \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}^T + \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & -1 \\ 5 & 2 & 4 \end{bmatrix}$$

b) $\frac{1}{3}BC$

NOT POSSIBLE

c) $(BA - C)^T$

$$= \left(\begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -1 & -1 \end{bmatrix} \right)^T$$

$$= \left(\begin{bmatrix} 4 & 5 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -1 & -1 \end{bmatrix} \right)^T = \begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix}^T = \begin{bmatrix} 4 & 1 \\ 2 & 8 \end{bmatrix}$$

Question 5. *(10 marks)* Find A given:

$$(5A-I)^{-1}=\begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$$

$$5A-I=\left[(5A-I)^{-1}\right]^{-1}=\begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}^{-1}=\frac{1}{-1}\begin{bmatrix}1 & -3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}$$

$$5A=\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}+I=\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 3\\ 1 & -1\end{bmatrix}$$

$$A=\frac{1}{5}\begin{bmatrix}0 & 3\\ 1 & -1\end{bmatrix}=\begin{bmatrix}0 & 3/5\\ 1/5 & -1/5\end{bmatrix}$$

Question 6. *(6 marks)* Given $n \times n$ invertable matrices A, B and C state whether the following statements are always true (T) or not (F).

a) $(B+C)A = BA + CA$ T
b) $(A^T)^T = A$ T
c) $(AB)^{-1} = B^{-1}A^{-1}$ T
d) $AB = BA$ F
e) $(B^5)^{-1} = (B^{-1})^5$ T
f) $(A+B)^{-1} = A^{-1} + B^{-1}$ F

Question 7. *(4 marks)* Determine which of the following are elementary matrices. Circle the elementary matrices:

$$A = \begin{bmatrix} 1 & 0 \\ \sqrt{5} & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bonus: *(1 mark)* Recommend an interesting item that you would not leave without when going on a long backpacking trip. Remember, space in your bag is an issue. If time permits, explain your answer.