Dawson College: Linear Algebra: 201-105-DW Section 05 February 16, 2009

Last Name: SOLUTIONS

First Name:

Student ID:

Test 1 (B)

Question 1. *(10 marks)*

a) Briefly state the three types of elementary row operations.

1) INTERCHANGE 2 ROWS
2) MULTIPLY A ROW BY A NON-ZERO CONSTANT
3) ADD A MULTIPLE OF A ROW TO ANOTHER ROW

b) Using elementary row operations, find the reduced row-echelon form of the following matrix.

$$\begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 2 & 1 & 2 \\ 2 & 4 & 0 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 4 & 0 & -2 \end{bmatrix} \xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -2 & -6 \end{bmatrix}$$

$$\xrightarrow{R_2 + 2R_1} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2. *(10 marks)* The following is an augmented matrix for a system of equations in row echelon form.

$$\begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 6 & 9 \\ 0 & 0 & a+5 & b-10 \end{bmatrix}$$

What are the values of a and b so that the system has:

a) no solutions?

$a+5=0$ AND $b-10=1$ ← ROW-ECHELON FORM

$\boxed{a=-5 \text{ AND } b=11}$

b) infinitely many solutions?

$a+5=0$ AND $b-10=0$

$\boxed{a=-5 \text{ AND } b=10}$

c) one solution?

$a+5=1$ AND b ANY NUMBER

$\boxed{a=-4}$

Question 3. *(10 marks)* Solve the following system of equations:

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & 3x_3 & - & 6x_4 & = & -2 \\ & & 2x_2 & + & 4x_3 & + & 4x_4 & = & 4 \\ x_1 & + & 3x_2 & + & 7x_3 & - & 2x_4 & = & 2 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 3 & -6 & -2 \\ 0 & 2 & 4 & 4 & 4 \\ 1 & 3 & 7 & -2 & 2 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 1 & 3 & -6 & -2 \\ 0 & 2 & 4 & 4 & 4 \\ 0 & 2 & 4 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 - R_2}$$

$$\begin{bmatrix} 1 & 1 & 3 & -6 & -2 \\ 0 & 2 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \cdot (\frac{1}{2})} \begin{bmatrix} 1 & 1 & 3 & -6 & -2 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - R_2}$$

$$\begin{bmatrix} 1 & 0 & 1 & -8 & -4 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

FREE VARIABLES:

$x_3 = s$, $x_4 = t$

- $x_2 + 2x_3 + 2x_4 = 2$

$x_2 = 2 - 2x_3 - 2x_4$

$= 2 - 2s - 2t$

- $x_1 + x_3 - 8x_4 = -4$

$x_1 = -4 - x_3 + 8x_4$

$= -4 - s + 8t$

SOLUTION SET

$(x_1, x_2, x_3, x_4) = (-4 - s + 8t,\ 2 - 2s - 2t,\ s,\ t)$

$s, t \in \mathbb{R}$

Question 4. *(10 marks)* Given:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Computing the following where possible. If not possible indicate so.

a) $A + C^T$ $= \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}^T$

$$= \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 1 & 0 \\ 4 & 3 \end{bmatrix}$$

b) $(B - CA)^T$ $= \left(\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 3 & 2 \end{bmatrix} \right)^T$

$$= \left(\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 2 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix}^T = \begin{bmatrix} -4 & -1 \\ -2 & -3 \end{bmatrix}$$

c) $5CB$ NOT POSSIBLE

Question 5. *(10 marks)* Find A given:

$$(3A - I)^{-1} = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix}$$

$$3A - I = \left[(3A - I)^{-1}\right]^{-1} = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 5 & -4 \end{bmatrix}$$

$$3A = \begin{bmatrix} -1 & 1 \\ 5 & -4 \end{bmatrix} + I = \begin{bmatrix} -1 & 1 \\ 5 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & -3 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} 0 & 1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 \\ 5/3 & -1 \end{bmatrix}$$

Question 6. *(6 marks)* Given $n \times n$ invertable matrices A, B, and C state whether the following statements are always true (T) or not (F).

a) $(A^T)^T = A$ T
b) $AB = BA$ F
c) $(AB)^{-1} = B^{-1}A^{-1}$ T
d) $(B+C)^{-1} = B^{-1} + C^{-1}$ F
e) $(A^{-1})^3 = (A^3)^{-1}$ T
f) $(A+B)C = AC + BC$ T

Question 7. *(4 marks)* Determine which of the following are elementary matrices. Circle the elementary matrices:

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & \sqrt{5} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bonus: *(1 mark)* Recommend an interesting item that you would not leave without when going on a long backpacking trip. Remember, space in your bag is an issue. If time permits, explain your answer.