Dawson College: **Calculus II**: 201-203-DW Section 04: Winter 2009 February 20, 2009

Name: SOLUTION

Student ID:

Test 1 (A)

No books are cell phones are allowed. You may use a nonprogrammable scientific calculator without graphing capabilities. Show all of your work and use correct notation.

Question 1. *(16 marks)* Find the indefinite integral:

(a) $\int \frac{(x^2+3)^2}{x^3}dx$ $= \int \frac{x^4+6x^2+9}{x^3}dx$

$(x^2+3)^2$

$= x^4+6x^2+9$

$= \int\left(x + \frac{6}{x} + \frac{9}{x^3}\right)dx = \int\left(x+\frac{6}{x}+9x^{-3}\right)dx$

$= \frac{x^2}{2} + 6\ln|x| + \frac{9x^{-2}}{-2} + C = \frac{x^2}{2} + 6\ln|x| - \frac{9}{2x^2} + C$

(b) $\int (3x^2-6x)e^{x^3-3x^2}dx$

LET $u = x^3 - 3x^2$

$du = (3x^2-6x)dx$

$dx = \frac{du}{3x^2-6x}$

$= \int \cancel{(3x^2-6x)}e^u \frac{du}{\cancel{3x^2-6x}}$

$= \int e^u du = e^u + C$

$= e^{x^3-3x^2} + C$

(c) $\int x^2 \cos(-2x^3+1)dx$

LET $u = -2x^3+1$

$du = -6x^2 dx$

$dx = \dfrac{du}{-6x^2}$

$= \int x^2 \cos u \dfrac{du}{-6x^2} = -\dfrac{1}{6}\int \cos u \, du$

$= -\dfrac{1}{6}\sin u + C = -\dfrac{1}{6}\sin(-2x^3+1) + C$

(d) $\int (x+2)(x-5)^7 dx$

LET $u = x-5$

$du = dx$

$x = u+5$

$= \int ((u+5)+2)(u)^7 du$

$= \int (u+7)u^7 du$

$= \int (u^8 + 7u^7) du = \dfrac{u^9}{9} + \dfrac{7u^8}{8} + C$

$= \dfrac{(x-5)^9}{9} + \dfrac{7(x-5)^8}{8} + C$

Question 2. *(6 marks)* Solve the initial value problem:

$$f'(x) = 9x^2 + x - 2, \quad f(1) = 5$$

$$f(x) = \int f'(x)\,dx = \int (9x^2 + x - 2)\,dx =$$

$$= \frac{9x^3}{3} + \frac{x^2}{2} - 2x + C = 3x^3 + \frac{x^2}{2} - 2x + C$$

$$5 = f(1) = 3(1)^3 + \frac{(1)^2}{2} - 2(1) + C = 3 + \frac{1}{2} - 2 + C$$

$$= \frac{3}{2} + C$$

$$5 - \frac{3}{2} = C \Rightarrow C = \frac{7}{2}$$

$$\therefore f(x) = 3x^3 + \frac{x^2}{2} - 2x + \frac{7}{2}$$

Question 3: *(10 marks)* Use the definition of the definite integral (Riemann Sum) to evaluate:

$$\int_0^3 (6x^2 - 2x)\,dx$$

Given:

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$x_k = 0 + k\left(\frac{3}{n}\right) = \frac{3k}{n}$$

$$f(x_k) = 6(x_k)^2 - 2(x_k) = 6\left(\frac{3k}{n}\right)^2 - 2\left(\frac{3k}{n}\right) = 54\frac{k^2}{n^2} - \frac{6k}{n}$$

$$f(x_k)\Delta x = \left(54\frac{k^2}{n^2} - \frac{6k}{n}\right)\left(\frac{3}{n}\right) = \frac{162k^2}{n^3} - \frac{18k}{n^2}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\frac{162k^2}{n^3} - \frac{18k}{n^2}\right) = \frac{162}{n^3}\sum_{k=1}^{n} k^2 - \frac{18}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{162}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{18}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= 27\left(\frac{2n^3+3n^2+n}{n^3}\right) - 9\left(\frac{n^2+n}{n^2}\right) = 27\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 9\cdot\left(1+\frac{1}{n}\right)$$

$$\int_0^3 (6x^2-2x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty}\left[27\left(2+\frac{3}{n}+\frac{1}{n^2}\right) - 9\left(1+\frac{1}{n}\right)\right]$$

$$= 27(2) - 9 = 45$$

Question 4. *(15 marks)* Evaluate the definite integral:

(a) $\int_2^3 (t-5)(3t+1)dt$

$$= \int_2^3 (3t^2 - 14t - 5)dx = \left[t^3 - 7t^2 - 5t\right]_2^3$$

$$= \left[(3)^3 - 7(3)^2 - 5(3)\right] - \left[(2)^3 - 7(2)^2 - 5(2)\right]$$

$$= 27 - 63 - 15 - 8 + 28 + 10$$

$$= -21$$

(b) $\int_{\frac{1}{2}}^2 \frac{\ln(2x)}{x}dx$

LET $u = \ln(2x)$

$du = \frac{1}{x}dx$

$x\,du = dx$

$x = \frac{1}{2} \Rightarrow u = \ln(1) = 0$

$x = 2 \Rightarrow u = \ln(4)$

$$= \int_0^{\ln 4} \frac{u}{\cancel{x}} \cancel{x} du = \int_0^{\ln 4} u\,du$$

$$= \left[\frac{u^2}{2}\right]_0^{\ln 4} = \frac{(\ln 4)^2}{2} - \frac{0^2}{2}$$

$$= \frac{(\ln 4)^2}{2}$$

(c) $\int_0^1 \frac{x}{\sqrt{7x^2+2}}dx$

LET $u = 7x^2+2$

$du = 14x\,dx$

$dx = \frac{du}{14x}$

$x=0 \Rightarrow u = 2$

$x=1 \Rightarrow u = 9$

$$= \int_2^9 \frac{\cancel{x}}{\sqrt{u}} \frac{du}{14\cancel{x}}$$

$$= \frac{1}{14}\int_2^9 u^{-\frac{1}{2}}\,du = \left[\frac{1}{14}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_2^9$$

$$= \frac{(9)^{\frac{1}{2}}}{7} - \frac{(2)^{\frac{1}{2}}}{7} = \frac{3}{7} - \frac{\sqrt{2}}{7} = \frac{3-\sqrt{2}}{7}$$

Question 5. *(10 marks)* The higher ups at Mattel Have determined that the daily marginal cost function associated with producing the Magic 8-Ball is given by:

$$C'(x) = 0.000009x^2 - 0.004x + 6$$

where $C'(x)$ is in dollars and x is the number of units produced. Since theft at the Magic 8-ball factory is an issue they pay a lot for security causing the daily fixed cost incurred in producing these Magic 8-Balls to be \$300.

(a) Find Mattel's daily total cost for producing the first 250 units. (You may use decimals in your answer.)

$$C(x) = \int (0.000009x^2 - 0.004x + 6)\,dx$$

$$= 0.000003x^3 - 0.002x^2 + 6x + C$$

$$C(0) = \$300 \Rightarrow \quad C = 300$$

$$\therefore \quad C(250) = 0.000003(250)^3 - 0.002(250) + 6(250) + 300$$

$$= \$1721.88$$

(b) Use the net change formula (definite integral) to find Mattel's daily total cost for producing the 301st through 400th unit. (You may use decimals in your answer.)

$$C(400) - C(300) = \int_{300}^{400} C'(x)\,dx$$

$$= \left[0.000003x^3 - 0.002x^2 + 6x\right]_{300}^{400}$$

$$= \left[0.000003(400)^3 - 0.002(400)^2 + 6(400)\right]$$

$$- \left[0.000003(300)^3 - 0.002(300)^2 + 6(300)\right]$$

$$= \$571.00$$