Dawson College: **Calculus II**: 201-203-DW Section 04: Winter 2009 February 20, 2009

Name: SOLUTIONS
Student ID: ______

Test 1 (B)

No books are cell phones are allowed. You may use a nonprogrammable scientific calculator without graphing capabilities. Show all of your work and use correct notation.

Question 1. *(16 marks)* Find the indefinite integral:

(a) $\int \frac{(t^2+5)^2}{t^3}dt = \int \frac{t^4+10t^2+25}{t^3}dt$

$$= \int \left(t + \frac{10}{t} + \frac{25}{t^3}\right) dt = \int \left(t + \frac{10}{t} + 25t^{-3}\right) dt$$

$$= \frac{t^2}{2} + 10\ln|t| + \frac{25t^{-2}}{-2} + C = \frac{t^2}{2} + 10\ln|t| - \frac{25}{2t^2} + C$$

(b) $\int (x-2)(x+3)^7 dx$

LET $u = x+3$
$du = dx$

$x = u-3$

$$= \int ((u-3)-2)u^7 du$$

$$= \int (u-5)u^7 du = \int (u^8 - 5u^7) du$$

$$= \frac{u^9}{9} - \frac{5u^8}{8} + C$$

$$= \frac{(x+3)^9}{9} - \frac{5(x+3)^8}{8} + C$$

(c) $\int (3x^2 - 10x)e^{x^3-5x^2}dx$

LET $u = x^3 - 5x^2$

$du = (3x^2 - 10x)dx$

$dx = \frac{du}{3x^2-10x}$

$= \int \cancel{(3x^2-10x)} e^u \frac{du}{\cancel{3x^2-10x}}$

$= \int e^u du = e^u + C$

$= e^{x^3-5x^2} + C$

(d) $\int x^2 \sin(-3x^3 + 5)dx$

LET $u = -3x^3 + 5$

$du = -9x^2 dx$

$dx = \frac{du}{-9x^2}$

$= \int \cancel{x^2} \sin u \frac{du}{-9\cancel{x^2}}$

$= \frac{-1}{9}\int \sin u\, du = -\frac{1}{9}(-\cos u) + C$

$= \frac{1}{9}\cos(-3x^3+5) + C$

Question 2. *(6 marks)* Solve the initial value problem:

$$f'(x) = 6x^2 + x - 3, \quad f(1) = 6$$

$f(x) = \int (6x^2 + x - 3)\,dx = 2x^3 + \frac{x^2}{2} - 3x + c$

$6 = f(1) = 2(1)^3 + \frac{1}{2} - 3(1) + c = 2 + \frac{1}{2} - 3 + c$

$= -1 + \frac{1}{2} + c$

$c = 6 + 1 - \frac{1}{2} = 7 - \frac{1}{2} = \frac{13}{2}$

$\therefore \quad f(x) = 2x^3 + \frac{x^2}{2} - 3x + \frac{13}{2}$

Question 3. *(10 marks)* Use the definition of the definite integral (Riemann Sum) to evaluate:

$$\int_0^4 (3x^2 - 4x)\,dx$$

Given:

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\Delta x = \frac{4-0}{n} = \frac{4}{n} \qquad x_k = 0 + k\left(\frac{4}{n}\right) = \frac{4k}{n}$$

$$f(x_k) = 3(x_k)^2 - 8(x_k) = 3\left(\frac{4k}{n}\right)^2 - 4\left(\frac{4k}{n}\right) = \frac{48k^2}{n^2} - \frac{16k}{n}$$

$$f(x_k)\Delta x = \left(\frac{48k^2}{n^2} - \frac{16k}{n}\right)\left(\frac{4}{n}\right) = 192\frac{k^2}{n^3} - 64\frac{k}{n^2}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(192\frac{k^2}{n^3} - \frac{64k}{n^2}\right) = \frac{192}{n^3}\sum_{k=1}^{n} k^2 - \frac{64}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{192}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{64}{n^2}\left(\frac{n(n+1)}{2}\right) = 32\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 32\left(1 + \frac{1}{n}\right)$$

$$\int_0^4 (3x^2 - 8x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \lim_{n\to\infty}\left[32\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 32\left(1 + \frac{1}{n}\right)\right]$$

$$= 32(2) - 32 = 32$$

Question 4. *(15 marks)* Evaluate the definite integral:

(a) $\int_2^4 (3x-1)(x+3)dx$ $= \int_2^4 (3x^2+8x-3)dx$

$= \left[\frac{3x^3}{3} + \frac{8x^2}{2} - 3x\right]_2^4 = \left[x^3+4x^2-3x\right]_2^4$

$= \left[(4)^3+4(4)^2-3(4)\right] - \left[(2)^3+4(2)^2-3(2)\right]$

$= 64 + 64 - 12 - 8 - 16 + 6 = 98$

(b) $\int_0^1 \frac{x}{\sqrt{6x^2+3}}dx$

LET $u = 6x^2+3$

$du = 12x\,dx$

$dx = \frac{du}{12x}$

$x=0 \Rightarrow u=3$

$x=1 \Rightarrow u=9$

$= \int_3^9 \frac{1}{\sqrt{u}}du$

$= \frac{1}{12}\int_3^9 u^{-\frac{1}{2}}du = \left[\frac{1}{12}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^9$

$= \frac{(9)^{\frac{1}{2}}}{6} - \frac{(3)^{\frac{1}{2}}}{6} = \frac{3-\sqrt{3}}{6}$

(c) $\int_{\frac{1}{6}}^{2} \frac{\ln(6x)}{x} dx =$

LET $u = \ln(6x)$

$du = \frac{1}{x} dx$

$x\,du = dx$

$x = \frac{1}{6} \Rightarrow u = \ln(1) = 0$

$x = 2 \Rightarrow u = \ln(12)$

$= \int_0^{\ln(12)} \frac{u}{\cancel{x}} \cancel{x} du$

$= \int_0^{\ln 12} u\, du$

$= \left[\frac{u^2}{2}\right]_0^{\ln 12} = \frac{(\ln 12)^2}{2} - \frac{0^2}{2} = \frac{(\ln 12)^2}{2}$

Question 5. *(10 marks)* The higher ups at the Ideal Toy Company have determined that the daily marginal cost function associated with producing Rubic's Cubes is given by:

$$C'(x) = 0.000006x^2 - 0.006x + 5$$

where $C'(x)$ is in dollars and x is the number of units produced. Since theft is an issue at the Rubic's Cube factory they pay a lot for security causing the daily fixed cost incurred in producing the Rubic's Cubes to be \$450.

(a) Find Mattel's daily total cost for producing the first 200 units. (You may use decimals in your answer.)

$$C(x) = \int C'(x)\,dx = \int (0.000006x^2 - 0.006x + 5)\,dx$$

$$= 0.000002x^3 - 0.003x^2 + 5x + c$$

$C(0) = 450 \Rightarrow c = 450$

$\therefore C(x) = 0.000002x^3 - 0.003x^2 + 5x + 450$

$$C(200) = 0.000002(200)^3 - 0.003(200)^2 + 5(200) + 450$$

$$= \$1346.00$$

(b) Use the net change formula (definite integral) to find the daily total cost for producing the 201st through 300th unit. (You may use decimals in your answer.)

$$C(300) - C(200) = \int_{200}^{300} C'(x)\,dx = \Big[0.000002x^3 - 0.003x^2 + 5x\Big]_{200}^{300}$$

$$= \left[0.000002(300)^3 - 0.003(300)^2 + 5(300)\right] - \left[0.000002(200)^3 - 0.003(200)^2 + 5(200)\right]$$

$$= \$388.00$$