# CLASS TEST II REVIEW

**The sections are: 1.7, 2.1, 2.2, 2.3, 2.4, 3.1, 3.2**

**(1) Find $A^2, A^{-2}, A^{-k}$ by inspection if:**

**(a)** $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

**(b)** $A = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

**(2) Find $a, b, c$ for which $A$ is symmetric:**

$$A = \begin{bmatrix} 5 & 2a+b & a-b-c \\ 4 & -1 & a+c \\ 0 & 0 & 2 \end{bmatrix}$$

**(3) If $A$ is symmetric show that $4A^2 - 2A + I$ is symmetric.**

**(4) If $A^T A = A$ then prove that $A$ is symmetric.**

**(5) Evaluate the following determinants by cofactor expansion:**

**(a)** $\begin{vmatrix} -3 & 1 & 4 \\ 0 & 5 & 0 \\ 2 & 0 & 2 \end{vmatrix}$ **(b)** $\begin{vmatrix} 2 & 4 & 3 \\ 2 & 6 & 5 \\ 1 & 0 & 1 \end{vmatrix}$ **(c)** $\begin{vmatrix} 4 & 1 & 3 \\ 0 & 1 & -2 \\ 1 & 2 & 5 \end{vmatrix}$

**(d)** $\begin{vmatrix} 1 & 7 & 2 & 0 \\ 2 & 5 & 5 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}$ **(e)** $\begin{vmatrix} 2 & k & 3 & 0 \\ k & 4 & 1 & 1 \\ 0 & -1 & 5 & 1 \\ 0 & 1 & 4 & 2 \end{vmatrix}$

**(6) Compute $A^{-1}$ by using the adjoint matrix:**

**(a)** $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 0\\ 3 & 0 & 3\end{bmatrix}$ **(b)** $A=\begin{bmatrix}1 & 3 & 5\\ 2 & 1 & 4\\ -1 & 6 & 2\end{bmatrix}$

**(c)** $A=\begin{bmatrix}3 & 0 & 0\\ 0 & 0 & -2\\ 0 & 8 & 0\end{bmatrix}$

**(7) Solve the following systems of equationsby using Cramer's Rule:**

**(a)** $\begin{cases}4x+2y=6\\ x-3y=5\end{cases}$ **(b)** $\begin{cases}x+y+z=2\\ 3x-y+z=5\\ 3x+2y+4z=0\end{cases}$

**(c)** $\begin{cases}x+y+z=0\\ 2x-y-z=1\\ y+2z=2\end{cases}$

**(8) Evaluate the determinants by row reduction to the row-echelon form:**

**(a)** $\begin{vmatrix}2 & 4 & 3\\ 2 & 6 & 5\\ 1 & 0 & 1\end{vmatrix}$ **(b)** $\begin{vmatrix}1 & 3 & -2\\ 4 & 2 & 5\\ 0 & 1 & 1\end{vmatrix}$ **(c)** $\begin{vmatrix}0 & 1 & 3 & 4\\ 2 & -7 & 2 & 4\\ -3 & 11 & 4 & 0\\ 1 & -3 & 2 & 2\end{vmatrix}$

**(9) Solve for $x$:**

$$\begin{vmatrix}2x+1 & 3 & 7\\ 0 & 5x & -1\\ 0 & 0 & 3\end{vmatrix}=0$$

**(10) If** $A=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$ **and** $\det(A)=7$ **then find:**

**(a)** $\det(-2A)$ **(b)** $\det(A^{-1})$

**(c)** $\det(A^{T})$ **(d)** $\det(5A^{-1})$ **(e)** $\det\begin{bmatrix}3a & 3b & 3c\\ g & h & i\\ d & e & f\end{bmatrix}$

**(11) For which $k$ does $A$ fail to be invertible?**

$$A = \begin{bmatrix} k+2 & -1 \\ 1 & k \end{bmatrix}$$

**(12) Find the number of inversions in the permutations of the set $(1,2,3,4,5)$:**

**(a)** $(3,4,1,2,5)$ **(b)** $(2,5,1,3,4)$ **(c)** $(1,3,5,4,2)$ **(d)** $(1,2,3,5,4)$

**(13) Evaluate the determinants by using signed elementary products:**

**(a)** $\begin{vmatrix} -3 & 1 & 4 \\ 0 & 5 & 0 \\ 2 & 0 & 2 \end{vmatrix}$ **(b)** $\begin{vmatrix} 2 & 4 & 3 \\ 2 & 6 & 5 \\ 1 & 0 & 1 \end{vmatrix}$ **(c)** $\begin{vmatrix} 4 & 1 & 3 \\ 0 & 1 & -2 \\ 1 & 2 & 5 \end{vmatrix}$

**(14) Solve for $x$:**

$$\begin{vmatrix} 3 & 0 & -1 \\ 1 & x & 2 \\ 1 & 2 & x+1 \end{vmatrix} = \begin{vmatrix} 2 & x \\ -1 & x+2 \end{vmatrix}$$

**(15) If $u = (1,7), v = (3,-2)$ then find:**

**(a)** $u+v$ **(b)** $2u-3v$ **(c)** $\|u-v\|$ **(d)** $\|u\|-\|v\|$

**(16) If $u = (1,0,-2), v = (3,2,4), w = (2,2,-1)$ then find:**

**(a)** $u-v$ **(b)** $2u+v$ **(c)** $3u+v-w$ **(d)** $\|u+v-2w\|$
**(e)** $\|u+v\|-\|2w\|$

**(17) Find the terminal point $Q$ of a vector $u$ with the initial point $P(4,-3,6)$ such that:**

**(a)** $u$ **has the same direction as** $v = (1,1,7)$;
**(b)** $u$ **is oppositely directed to** $v = (1,1,7)$

**(18) Find the initial point $P$ of a vector $u$ with the terminal point $Q(3,5,5)$ such that:**

**(a)** $u$ **has the same direction as** $v = (6,-2,1)$;
**(b)** $u$ **is oppositely directed to** $v = (6,-2,1)$

**(19) Findthe distance between $P$ and $Q$:**
**(a)** $P(1,7), Q(2,3)$ **(b)** $P(1,0,6), Q(4,3,-2)$ **(c)** $P(7,-4,5), Q(8,-2,-3)$

**ANSWERS:**

**(1)** **(a)** $A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}, A^{-2} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}, A^{-k} = \begin{bmatrix} \frac{1}{2^k} & 0 \\ 0 & (-1)^k \end{bmatrix}$

**(b)** $A^2 = \begin{bmatrix} \frac{1}{25} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{9} \end{bmatrix}, A^{-2} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}, A^{-k} = \begin{bmatrix} 5^k & 0 & 0 \\ 0 & 2^k & 0 \\ 0 & 0 & 3^k \end{bmatrix}$

**(2)** $a = 1$, $b = 2$, $c = -1$

**(5)** **(a)** -70 **(b)** 6 **(c)** 31 **(d)** .66 **(e)** $-6k^2 - 9k + 36$

**(6)** **(a)** $A^{-1} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ 1 & 1 & -1 \\ -\frac{1}{2} & -1 & \frac{5}{6} \end{bmatrix}$ **(b)** $A^{-1} = \frac{1}{19}\begin{bmatrix} -22 & 24 & 7 \\ -8 & 7 & 6 \\ 13 & -9 & -5 \end{bmatrix}$

**(c)** $A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{8} \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$

**(7)** **(a)** $x = 2, y = -1$ **(b)** $x = \frac{7}{4}, y = -\frac{17}{24}, z = -\frac{23}{24}$ **(c)** $x = \frac{1}{3}, y = -\frac{8}{3}, z = \frac{7}{3}$

**(8)** **(a)** 6 **(b)** -23 **(c)** 18

**(9)** $x = 0, y = -\frac{1}{2}$

**(10)** **(a)** -56 **(b)** $\frac{1}{7}$ **(c)** 7 **(d)** $\frac{125}{7}$ **(e)** -21

**(11)** $k = -1$

**(12)** **(a)** 4 **(b)** 4 **(c)** 4 **(d)** 1

**(13)** **(a)** -70 **(b)** 6 **(c)** 31

**(14)** $x_{1,2} = \frac{-1 \pm \sqrt{217}}{6}$

**(15)** **(a)** $(4,5)$ **(b)** $(-7,20)$ **(c)** $\sqrt{85}$ **(d)** $5\sqrt{2} - \sqrt{13}$

**(16)** **(a)** $(-2,-2,-6)$ **(b)** $(5,2,0)$ **(c)** $(4,0,-1)$ **(d)** $2\sqrt{5}$ **(e)** $2\sqrt{6}-6$

**(17)** **(a)** $Q(5,-2,13)$ **(b)** $Q(3,-4,-1)$

**(18)** **(a)** $P(5,-2,13)$ **(b)** $P(9,3,6)$

**(19)** **(a)** $\sqrt{17}$ **(b)** $\sqrt{85}$ **(c)** $\sqrt{69}$