Dawson College: **Calculus II**: 201-203-DW Section 04: Winter 2009

March 27, 2009

Name: SOLUTION

Student ID:

Test 2

No books or cell phones are allowed. You may use a nonprogrammable scientific calculator without graphing capabilities. Show all of your work and use correct notation.

Question 1. *(4 marks)* Find the average of $f(x) = 6x^2 - 4x + 1$ on the interval $[1,5]$.

$$\text{AVERAGE} = \frac{1}{5-1}\int_1^5 (6x^2 - 4x + 1)\,dx$$

$$= \frac{1}{4}\left[2x^3 - 2x^2 + x\right]_1^5$$

$$= \frac{1}{4}\left[\left(2(5)^3 - 2(5)^2 + 5\right) - \left(2(1)^3 - 2(1)^2 + 1\right)\right]$$

$$= \frac{1}{4}\left[250 - 50 + 5 - 2 + 2 - 1\right]$$

$$= \frac{1}{4}(204)$$

$$= 51$$

Question 2. *(10 marks)* Let $f(x) = x^2 - 2x$ and $g(x) = x + 4$.

(a) Find the points intersection for these two functions.

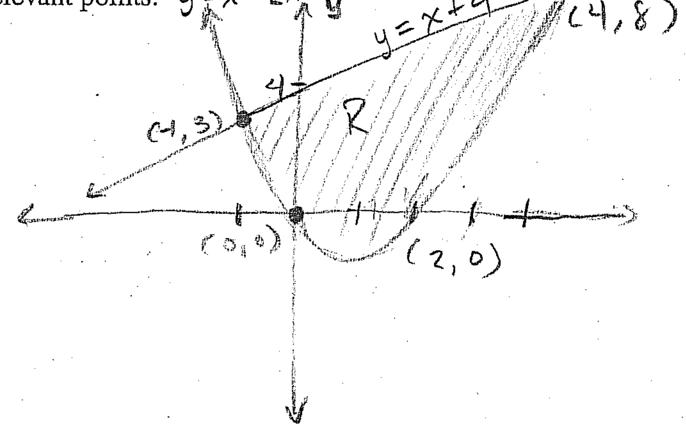
$$2x^2 - 2x = x + 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4, \ -1$$

(b) Sketch the graphs of $f(x)$ and $g(x)$. Shade the region R between the two curves. Make sure to label all relevant points.



(c) Find the area of R, the region between $f(x)$ and $g(x)$.

$$\text{Area of } R = \int_{-1}^{4} \left[(x+4) - (x^2 - 2x)\right] dx$$

$$= \int_{-1}^{4} (-x^2 + 3x + 4)\, dx = \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^{4}$$

$$= \left[\frac{-(4)^3}{3} + \frac{3(4)^2}{4} + 4(4) \right] - \left[-\frac{(-1)^3}{3} + \frac{3(-1)^2}{2} + 4(-1) \right]$$

$$= \frac{125}{6} \text{ units}^2$$

Question 3. *(10 marks)* The demand function for a certain brand of burnable DVDs is given by

$$p = -0.01x^2 - 0.2x + 8$$

where p is the unit price in dollars and x is the quantity demanded each week measured in units of a thousand. The wholesale market price is set at \$5 per DVD.

(a) Determine the consumer's surplus.

$$\boxed{\begin{aligned} 5 &= -0.01x^2 - 0.2x + 8 \\ 0.01x^2 &+ 0.2x - 3 = 0 \\ x^2 &+ 20x - 300 = 0 \\ (x+30)&(x-10) = 0 \\ \bar{x} &= 10, \cancel{-30} \end{aligned}}$$

$$CS = \int_0^{\bar{x}} D(x)\,dx - \bar{p}\bar{x}$$

$$= \int_0^{10} (-0.01x^2 - 0.2x + 8)\,dx - (5)(10)$$

$$= \left[-\frac{0.01x^3}{3} - \frac{0.2}{2}x^2 + 8x\right]_0^{10} - 50 =$$

$$= \left[-\frac{0.01(10)^3}{3} - 0.1(10)^2 + 8(10)\right] - [0] - 50 = 16.\bar{6}$$

$$\therefore CS = \$16.666.67$$

(b) Determine the producer's surplus if the supply function for the DVDs is

$$p = 0.01x^2 + 0.1x + 3$$

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\,dx = (5)(10) - \int_0^{10} (0.01x^2 + 0.1x + 3)\,dx$$

$$= 50 - \left[\frac{0.01x^3}{3} + \frac{0.1}{2}x^2 + 3x\right]_0^{10}$$

$$= 50 - \left(\left[\frac{0.01(10)^3}{3} + \frac{0.1}{2}(10)^2 + 3(10)\right] - [0]\right)$$

$$= 11.\bar{6}$$

$$\therefore PS = \$11\,666.67$$

Question 4. Find each integral (Definite or indefinite):
(a) *(5 marks)*

$$\int 6xe^{-3x}dx$$

$u = 6x$ $\quad dv = e^{-3x}dx$

$du = 6dx$

$= uv - \int v\,du$

$= -2xe^{-3x} - \int \left(-\frac{1}{3}e^{-3x}\right)6dx$

$= -2xe^{-3x} + \int 2e^{-3x}dx$

$= -2xe^{-3x} - \frac{2}{3}e^{-3x} + c$

(b) *(5 marks)*

$$\int \frac{\ln x}{x^3}dx$$

$u = \ln x$ $\quad dv = \frac{1}{x^3}dx$

$du = \frac{1}{x}dx$ $\quad v = -\frac{1}{2}x^{-2}$

$= -\frac{1}{2}x^{-2}\ln x \quad \int \frac{1}{x}\left(-\frac{1}{2}x^{-2}\right)dx$

$= -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\int x^{-3}dx$

$= -\frac{1}{2}x^{-2}\ln x - \frac{1}{4}x^{-2} + c$

$= -\frac{\ln x}{2x} - \frac{1}{4x^2} + c$

(c) *(5 marks)*

$$\int_1^2 4x^3 \ln(x^3)dx$$

$u = \ln(x^3)$ $\quad dv = 4x^3 dx$

$du = \frac{1}{x^3} 3x^2 dx$ $\quad v = x^4$

$$= [uv]_1^2 - \int_1^2 v\,du$$

$$= \left[x^4 \ln x^3\right]_1^2 - \int_1^2 3x^3 dx = \left[x^4 \ln x^3\right]_1^2 - \left[\tfrac{3}{4}x^4\right]_1^2$$

$$= (2^4 \ln 8 - 0) - (12 - \tfrac{3}{4})$$

$$= 16\ln 8 - \frac{45}{4}$$

(d) *(6 marks)*

LET $I = \int e^x \sin x\, dx$

LET $u = \sin x$ $\quad dv = e^x dx$

$du = \cos x dx$ $\quad v = e^x$

$$= uv - \int v\,du$$

LET $U = \cos x$ $\quad dV = e^x dx$

$dU = -\sin x dx$ $\quad V = e^x$

$$= e^x \sin x - \int e^x \cos x\, dx$$

$$= e^x \sin x - \left(UV - \int V dU\right) = e^x \sin x - e^x \cos x + \int e^x(-\sin x)dx$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

$$= e^x \sin x - e^x \cos x - I$$

$$\therefore \quad 2I = e^x \sin x - e^x \cos x$$

$$I = \underline{\frac{e^x \sin x - e^x \cos x}{2}} + C$$

(e) *(6 marks)*

$$I = \int \frac{3x^2-x-1}{(x-2)(x+1)^2}dx$$

$$\frac{3x^2-x-1}{(x-2)(x+1)^2} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$3x^2-x-1 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$

$\underline{x=-1}$

$3+1-1 = -3C$

$C = -1$

$\underline{x=2}$

$12-2-1 = A(2+1)^2$

$9 = 9A$

$A = 1$

$\underline{x=0}$

$-1 = 1(1)^2 + B(-2)(1) - 1(-2)$

$-4 = -2B$

$B = 2$

$$\therefore I = \int\left(\frac{1}{x-2} + \frac{2}{x+1} - \frac{1}{(x+1)^2}\right)dx$$

$$= \ln|x-2| + 2\ln|x+1| + \frac{1}{x+1} + C$$

(f) *(6 marks)*

$$I = \int \frac{2x^3 - 7x^2 + 7x + 5}{2x - 1} dx$$

$$
\begin{array}{r|l}
 & x^2 \quad -3x \quad +2 \\
\hline
2x-1 & 2x^3 - 7x^2 + 7x + 5 \\
 & -(2x^3 - x^2) \\
\hline
 & \quad -(-6x^2 + 3x) \\
\hline
 & \qquad\quad 4x + 5 \\
 & \qquad -(4x - 2) \\
\hline
 & \qquad\qquad 7
\end{array}
$$

$$\therefore \int \frac{2x^3 - 7x^2 + 7x + 5}{2x - 1} dx = \int \left(x^2 - 3x + 2 + \frac{7}{2x-1} \right) dx$$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x + \frac{7}{2} \ln|2x - 1| + C$$

Question 5. *(7 marks)*
Use Simpson's rule with $n = 6$ to approximate the following. Round your final answer to 3 decimal places.

$$\int_2^5 \frac{\sqrt{x}}{\ln x} dx$$

$$\Delta x = \frac{5-2}{6} = \frac{1}{2}$$

$x_0 = 2$, $x_1 = 2.5$, $x_2 = 3$, $x_3 = 3.5$, $x_4 = 4$, $x_5 = 4.5$
$x_6 = 5$

$$\int_1^4 \frac{\sqrt{x}}{\ln x} dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right]$$

$$= \frac{1}{6}\left[\frac{\sqrt{2}}{\ln 2} + 4\frac{\sqrt{2.5}}{\ln 2.5} + 2\frac{\sqrt{3}}{\ln 3} + 4\frac{\sqrt{3.5}}{\ln 3.5} + 2\frac{\sqrt{4}}{\ln 4} + 4\frac{\sqrt{4.5}}{\ln 4} + \frac{\sqrt{5}}{\ln 5}\right]$$

≈ 4.664