CLASS TEST III   REVIEW

The sections are: 3.3, 3.4, 3.5,

(1) **If** $u = (1,1,3), v = (2,0,1), w = (0,-1,5)$ **then find:**

  (a) $2u + w$ (b) $2(u+v) - 3(v-2w)$ (c) $(u \cdot v)w$ (d) $u \cdot (v \times w)$
  (e) $\|v\|(u \cdot w)$ (f) $\|u \times (v+w)\|$

(2) **Let** $u = (1,1,3), v = (2,0,1), w = (0,-1,5)$. **Compute the following:**

  (a) $u - 2v + w$ (b) $w \cdot u - \|5v\|$ (c) $(2u-v) \times w$ (d) $(u \times v) \times w$
  (e) $(u \times w) \times (u \times v)$ (f) $(v \times w) \cdot u$

(3) **Find the projection of** u **onto** a :

(a) $u = (6,5)$, $a = (-3,4)$
(b) $u = (1,2)$, $a = (7,-3)$
(c) $u = (3,0,2)$, $a = (1,1,3)$

(4) **Find the vector component of** u **orthogonal to** a :

(a) $u = (4,1)$, $a = (2,-3)$
(b) $u = (5,-2)$, $a = (1,1)$
(c) $u = (1,5,-5)$, $a = (1,2,3)$

(5) **Consider the points** $A(2,6), B(3,7), C(3,8)$. **Compute the following:**

(a) cosine of the angle between $\overrightarrow{BA}$ and $\overrightarrow{BC}$
(b) cosine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$
(c) cosine of the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$

(6) **Calculate the distance between the given point and line:**

(a) $x - 2y + 3 = 0$, $(2,1)$
(b) $5x + 12y - 2 = 0$, $(2,-1)$

(7) Find the area of the triangle PQR :

(a) $P(1,3,2), Q(2,3,1), R(2,2,3)$
(b) $P(3,-3,1), Q(1,-3,2), R(5,-2,-1)$
(c) $P(3,0,-1), Q(2,2,2), R(4,2,3)$

(8) Find the area of the triangle determined by u and v :

(a) $u = (1,2,4), \ v = (3,1,2)$
(b) $u = (1,-1,2), \ v = (2,0,3)$

(9) Find the area of the parallelogram determined by u and v :

(a) $u = (1,0,-2), \ v = (0,3,2)$
(b) $u = (2,0,2), \ v = (0,2,1)$

(10) Determine whether the lines are parallel:
(x, y, z) = (2+t, 1+2t, 1+t) and (x, y, z) = (4+2t, 2+4t, 4+2t)

(11) Determine whether the planes are parallel:

(a) $4x + y - 7z = 1, \ 4x + y - 7z = 0$
(b) $-x + 2y - z = 1, \ 3x + 6y + 3z = 5$

(12) Determine whether the planes are perpendicular:

(a) $2x - 3y + z = 0, \ 4x - 6y + 2z = 3$
(b) $x - 3y + 5z = 2, \ 4x + 3y + z = 0$

(13) Determine whether the line and plane are parallel:

(x, y, z) = (2+4t, 1-3t, t),   x + y − z = 5

(14) Determine whether the line and plane are perpendicular:
(x, y, z) = (3+2t, 14t, 1+ 12t),   x + 7y = 2 + 6z

(15) Give the point of intersection, if any, of the line and plane:

(a) $(x, y, z) = (4+2t, 7-t, 3t),\ 2x - y - z = 7$
(b) $(x, y, z) = (t-3, 2t, t+5),\ x - 4y - z = 0$

(16) Find a parametric equation for the line of intersection of the planes:

$x + 2y - z = 1,\ 2x - y + 3z = 7$

(17) Find a parametric equation for the line passing through the given points:

$P(1,6,3), Q(2,7,1)$

(18) Find a general form of the equation of the plane passing through P and having n as normal:

(a) $P(2,3,4), n = (6,1,6)$
(b) $P(8,-8,2), n = \left(-\dfrac{2}{3}, \dfrac{1}{3}, \dfrac{2}{3}\right)$

(19) Find the equation of the plane passing through the given points:

(a) $P(3,1,1), Q(1,6,7), R(4,2,2)$
(b) $P(3,4,4), Q(0,1,-11), R(1,3,-17)$

(20) Find an equation of the plane through $P(1,1,3)$ that is perpendicular to the line $(x, y, z) = (2 - 3t, 1 + t, 2t)$.

(21) Find an equation of the plane through $P(2,7,-1)$ that is parallel to the plane $4x - y + 3z = 3$.

(22) Find an equation of the plane that contains the line $(x, y, z) = (3 + t, 5, 5 + 2t)$, and is perpendicular to the plane $x + y + z = 4$.

(23) Find an equation of the plane through $P(1,4,4)$ that contains the line of intersection of the planes $x - y + 3z = 5$ and $2x + 2y + 7z = 0$.

(24) Find an equation of the line through $P(2,-3,0)$ that is parallel to the planes $2x + 2y + z = 2$ and $x - 3y = 5$.

(25) **Find an equation of the plane through** $P(-1,7,4)$ **that is perpendicular to the planes** $3x + y - z = 5$ **and** $11x + 2y + 3z = 0$.

(26) **Calculate the distance between the given point and the plane:**

(a) $2x - 3y + 6z + 4 = 0$, $(1,1,-1)$

(b) $z = 2x + 2y + 8$, $(11,-2,3)$

<u>ANSWERS</u>

1) a) $(2, 1, 11)$
   b) $(0, -4, 35)$
   c) $(0, -5, 25)$
   d) -18
   e) $14\sqrt{5}$
   f) $3\sqrt{10}$

2) a) $(-2, -3, -6)$
   b) -17
   c) $(9, 0, 3)$
   d) $(0, -6, 0)$
   e) $(12, -12, 12)$
   f) -12

3) a) $\left(\frac{-6}{25}, \frac{8}{25}\right)$
   b) $\left(\frac{7}{58}, \frac{-3}{58}\right)$
   c) $\left(\frac{9}{11}, \frac{9}{11}, 3\right)$

4) a) $\left(\frac{42}{13}, \frac{28}{13}\right)$
   b) $\left(\frac{7}{2}, \frac{-7}{2}\right)$
   c) $\left(\frac{9}{7}, \frac{39}{7}, \frac{-29}{7}\right)$

5) a) $\frac{-\sqrt{2}}{2}$
   b) $\frac{3\sqrt{10}}{10}$
   c) $\frac{2\sqrt{5}}{5}$

6) a) $\frac{3\sqrt{5}}{5}$
   b) $\frac{4}{13}$

7) a) $\frac{\sqrt{6}}{2}$
   b) $3/2$
   c) $\frac{\sqrt{69}}{2}$

8) a) $\frac{5\sqrt{5}}{2}$
   b) $\frac{\sqrt{14}}{2}$

9) a) 7
   b) 6

10) YES

11) a) YES
    b) NO

12) a) NO
    b) YES

13) YES

14) NO

15) a) $(10, 4, 9)$
    b) $(-4, -2, -4)$

16) $(x, y, z) = (3-t, -1+t, t)$ $t \in \mathbb{R}$

17) $(x, y, z) = (1+t, 6+t, 3-2t)$ $t \in \mathbb{R}$

18) a) $6x + y + 6z = 39$
    b) $-2x + y + 2z = -20$

19) a) $-(x-3) + 8(y-1) - 7(z-1) = 0$

b) $16(x-3) - 11(y-4) - (z-4) = 0$

20) $-3x + y + 2z = 4$

21) $4x - y + 3z = -2$

22) $-2x + y + z = 4$

23) $15x - 23y + 43z = 95$

24) $(x, y, z) = (2 + 3t, -3 + t, -8t) \quad t \in \mathbb{R}$

25) $x - 4y - z = -33$

26) a) $\frac{3}{7}$ \qquad b) $\frac{23}{3}$