# Probability & Statistics FORMULAS

Sample mean: $\bar{x} = \dfrac{\sum x}{n}$ or $\bar{x} = \dfrac{\sum M \cdot f}{n}$ where $M$ = class mark

Total variation in $x$: $SS(x) = \sum (x_i - \bar{x})^2 = \sum (x - \bar{x})^2 = \sum x^2 - \dfrac{(\sum x)^2}{n}$

Sample standard deviation: $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - \dfrac{(\sum x)^2}{n}}{n-1}}$

or $s = \sqrt{\dfrac{\sum (M - \bar{x})^2 \cdot f}{n-1}} = \sqrt{\dfrac{\sum M^2 \cdot f - \dfrac{(\sum M \cdot f)^2}{n}}{n-1}}$

Standard scores: $z = \dfrac{x - \bar{x}}{s}$ or $z = \dfrac{x - \mu}{\sigma}$

Rules of probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad P(A \cap B) = P(A) \cdot P(B \mid A)$$

Combinations: ${}_nC_x = \dfrac{n!}{x! \cdot (n-x)!} = \dbinom{n}{x} = C_x^n$

Correlation & regression $SS(x) = \sum x^2 - \dfrac{(\sum x)^2}{n}$ $SS(y) = \sum y^2 - \dfrac{(\sum y)^2}{n}$

$SS(xy) = \sum xy - \dfrac{(\sum x)(\sum y)}{n}$ $\hat{y} = b_0 + b_1 \cdot x$ = line of best fit

$b_1 = \dfrac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \dfrac{SS(xy)}{SS(x)}$ $b_0 = \bar{y} - b_1 \cdot \bar{x}$

$$r = \frac{\sum (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\left(\sum (x_i - \bar{x})^2\right) \cdot \left(\sum (y_i - \bar{y})^2\right)}} = \frac{SS(xy)}{\sqrt{SS(x) \cdot SS(y)}}$$

Probability distributions: general case

expected value $\mu = \sum x \cdot P(x)$

variance $\sigma^2 = \sum (x - \mu)^2 \cdot P(x) = \sum x^2 \cdot P(x) - \mu^2$

Binomial probability distribution: $P(x) = {}_nC_x \cdot p^x \cdot (1-p)^{n-x}$

expected value $\mu = n \cdot p$ variance $\sigma^2 = n \cdot p \cdot (1-p)$

Poisson probability distribution: $P(x) = \dfrac{\mu^x \cdot e^{-\mu}}{x!}$

Hypergeometric probability distribution: $P(x) = \dfrac{({}_rC_x) \cdot ({}_{N-r}C_{n-x})}{{}_NC_n}$

Sampling distribution of sample means $\bar{x}$:

$$\mu_{\bar{x}} = \mu_x = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{\sigma}{\sqrt{n}} \qquad z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

Margin of error: $E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$ or $E = t_{\alpha/2} \cdot \dfrac{s}{\sqrt{n}}$ with $df = n-1$

Sampling distribution of sample proportions $p' = \dfrac{x}{n}$: $\mu_{p'} = p$ $\sigma_{p'} = \sqrt{\dfrac{p \cdot (1-p)}{n}}$

$z = \dfrac{p'-p}{\sigma_{p'}}$ margin of error $= E = z_{\alpha/2} \cdot \sqrt{\dfrac{p' \cdot (1-p')}{n}}$

Test of Hypothesis for one mean $z = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

or $t = \dfrac{\bar{x}-\mu_0}{s/\sqrt{n}}$ with $df = n-1$ .

Difference of means

$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 =$ "true" difference of means

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}} =$ standard deviation

margin of error $= E = z_{\alpha/2} \cdot \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$.

$$z = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \qquad \text{or} \qquad t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

with degrees of freedom $df = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{s_1^4}{n_1^2 \cdot (n_1 - 1)} + \dfrac{s_2^4}{n_2^2 \cdot (n_2 - 1)}}$

margin of error = $E = t_{\alpha/2} \bullet \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

Paired difference $d = x_1 - x_2$

$$\mu_{\bar{d}} = \mu_d \qquad \sigma_{\bar{d}} = \frac{\sigma_d}{\sqrt{n}}$$

$E = t_{\alpha/2} \bullet \dfrac{s_d}{\sqrt{n}}$ and $t = \dfrac{\bar{d} - d_o}{s_d / \sqrt{n}}$ with $df = n - 1$

where $d_o$ is the value claimed in $H_o$

Difference of Proportions

$$p_1' = \frac{x_1}{n_1}, \quad p_2' = \frac{x_2}{n_2}, \qquad \mu_{p_1' - p_2'} = p_1 - p_2$$

$$\sigma_{p_1' - p_2'} = \sqrt{\frac{p_1 \cdot (1 - p_1)}{n_1} + \frac{p_2 \cdot (1 - p_2)}{n_2}}$$

margin of error $= E = z_{\alpha/2} \bullet \sqrt{\dfrac{p_1' \cdot (1 - p_1')}{n} + \dfrac{p_2' \cdot (1 - p_2')}{n_2}}$

pooled proportion $= p_p' = \dfrac{x_1 + x_2}{n_1 + n_2}$

$$z = \frac{p_1' - p_2' - (0)}{\sqrt{p_p' \cdot (1 - p_p') \cdot \left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

Chi-squared

$$\chi^2 = \sum_{j=1}^{k} \frac{(f_j - e_j)^2}{e_j} \quad \text{with } df = k - 1$$

or $$\chi^2 = \sum_{i}\sum_{j} \frac{(f_{i,j} - e_{i,j})^2}{e_{i,j}} \quad \text{with } df = (r-1) \bullet (c-1)$$

$$\text{expected value} = \frac{(\textbf{row total}) \bullet (\textbf{column total})}{\textbf{grand total}}$$

testing variance $$\chi^2 = \frac{(n-1)\, s^2}{\sigma^2} \quad \text{with } df = n-1$$

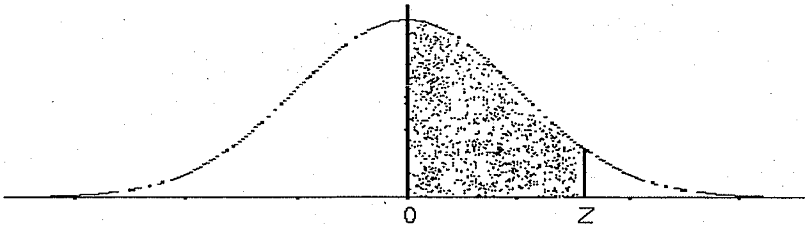
Continuous Distributions

$$P(\, a \le x \le b \,) = \int_a^b f(x)\, dx$$

$$P(\, x \le b \,) = P(\, x < b \,) = \int_{-\infty}^{b} f(x)\, dx, \qquad P(\, x \ge a \,) = P(x > a\,) = \int_a^{\infty} f(x)\, dx$$

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

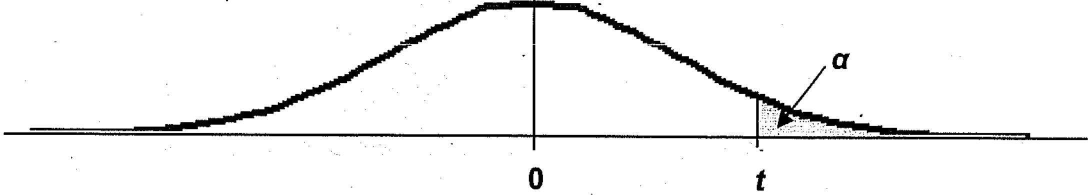
$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\, dx = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$$




**Areas under the standard normal curve.**

| z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 4.0 | 0.49997 | | | | | | | | | |
| 4.5 | 0.499997 | | | | | | | | | |
| 5.0 | 0.4999997 | | | | | | | | | |

## Student's *t* – distribution




| | α | | | | | | α | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| df | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 | df | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 | 51 | 1.298 | 1.675 | 2.008 | 2.402 | 2.676 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 52 | 1.298 | 1.675 | 2.007 | 2.400 | 2.674 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 53 | 1.298 | 1.674 | 2.006 | 2.399 | 2.672 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 54 | 1.297 | 1.674 | 2.005 | 2.397 | 2.670 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 55 | 1.297 | 1.673 | 2.004 | 2.396 | 2.668 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 56 | 1.297 | 1.673 | 2.003 | 2.395 | 2.667 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 57 | 1.297 | 1.672 | 2.002 | 2.394 | 2.665 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 58 | 1.296 | 1.672 | 2.002 | 2.392 | 2.663 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 59 | 1.296 | 1.671 | 2.001 | 2.391 | 2.662 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 61 | 1.296 | 1.670 | 2.000 | 2.389 | 2.659 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 62 | 1.295 | 1.670 | 1.999 | 2.388 | 2.657 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 63 | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 64 | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 65 | 1.295 | 1.669 | 1.997 | 2.385 | 2.654 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 66 | 1.295 | 1.668 | 1.997 | 2.384 | 2.652 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 67 | 1.294 | 1.668 | 1.996 | 2.383 | 2.651 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 68 | 1.294 | 1.668 | 1.995 | 2.382 | 2.650 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 69 | 1.294 | 1.667 | 1.995 | 2.382 | 2.649 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 71 | 1.294 | 1.667 | 1.994 | 2.380 | 2.647 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 72 | 1.293 | 1.666 | 1.993 | 2.379 | 2.646 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 73 | 1.293 | 1.666 | 1.993 | 2.379 | 2.645 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 74 | 1.293 | 1.666 | 1.993 | 2.378 | 2.644 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 75 | 1.293 | 1.665 | 1.992 | 2.377 | 2.643 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 76 | 1.293 | 1.665 | 1.992 | 2.376 | 2.642 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 77 | 1.293 | 1.665 | 1.991 | 2.376 | 2.641 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 78 | 1.292 | 1.665 | 1.991 | 2.375 | 2.640 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 79 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 | 81 | 1.292 | 1.664 | 1.990 | 2.373 | 2.638 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 | 82 | 1.292 | 1.664 | 1.989 | 2.373 | 2.637 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 | 83 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 | 84 | 1.292 | 1.663 | 1.989 | 2.372 | 2.636 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 | 85 | 1.292 | 1.663 | 1.988 | 2.371 | 2.635 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 | 86 | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 |
| 37 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 | 87 | 1.291 | 1.663 | 1.988 | 2.370 | 2.634 |
| 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 | 88 | 1.291 | 1.662 | 1.987 | 2.369 | 2.633 |
| 39 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 | 89 | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 |
| 41 | 1.303 | 1.683 | 2.020 | 2.421 | 2.701 | 91 | 1.291 | 1.662 | 1.986 | 2.368 | 2.631 |
| 42 | 1.302 | 1.682 | 2.018 | 2.418 | 2.698 | 92 | 1.291 | 1.662 | 1.986 | 2.368 | 2.630 |
| 43 | 1.302 | 1.681 | 2.017 | 2.416 | 2.695 | 93 | 1.291 | 1.661 | 1.986 | 2.367 | 2.630 |
| 44 | 1.301 | 1.680 | 2.015 | 2.414 | 2.692 | 94 | 1.291 | 1.661 | 1.986 | 2.367 | 2.629 |
| 45 | 1.301 | 1.679 | 2.014 | 2.412 | 2.690 | 95 | 1.291 | 1.661 | 1.985 | 2.366 | 2.629 |
| 46 | 1.300 | 1.679 | 2.013 | 2.410 | 2.687 | 96 | 1.290 | 1.661 | 1.985 | 2.366 | 2.628 |
| 47 | 1.300 | 1.678 | 2.012 | 2.408 | 2.685 | 97 | 1.290 | 1.661 | 1.985 | 2.365 | 2.627 |
| 48 | 1.299 | 1.677 | 2.011 | 2.407 | 2.682 | 98 | 1.290 | 1.661 | 1.984 | 2.365 | 2.627 |
| 49 | 1.299 | 1.677 | 2.010 | 2.405 | 2.680 | 99 | 1.290 | 1.660 | 1.984 | 2.365 | 2.626 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| | | | | | | z | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |