Dawson College: **Calculus I**: 201-NYA-05-S6: Winter 2009.

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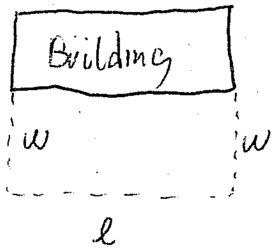
Quiz 7

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* pg. 723 #18 A rectangular storage area is to be constructed along the side of a tall building. A security fence is required along the remaining three sides of the area. What is the maximum area that can be enclosed with 800 m of fencing?



① $A = lw$

② $800 = 2w + l$

$800 - 2w = l$

sub ② into ①

$A = (800 - 2w)w$

$A = 800w - 2w^2$

$A' = 800 - 4w$

Lets. find the critical points

$0 = 800 - 4w$

$w = 200$

Lets verify that it is a max

$A'' = -4 < 0$ $\therefore$ max. by 2^{nd} derivative test

$\therefore l = 800 - 2w$

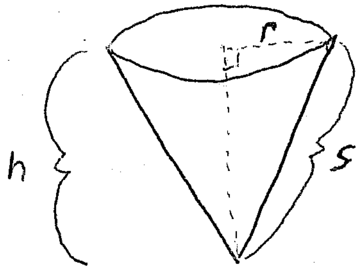
$= 800 - 2(200)$

$= 400$

$\therefore$ $w = 200\,m$

$l = 400\,m$

Question 2. *(5 marks)* pg. 724 #36 A cone-shaped paper cup is to hold 100 cm^3 of H_2O. Find the height and radius of the cup that can be made from the least amount of paper.



$s^2 = r^2 + h^2$

$s = \sqrt{r^2 + h^2}$

$V = \frac{1}{3}\pi r^2 h \Rightarrow 100 = \frac{1}{3}\pi r^2 h$

$h = \frac{300}{\pi r^2}$

$A = \pi r s + \cancel{\pi r^2}$ (no top)

$A = \pi r s$

$A = \pi r \sqrt{r^2 + h^2}$

$A = \pi r \sqrt{r^2 + \left(\frac{300}{\pi r^2}\right)^2}$

$A = \sqrt{\pi^2 r^2 \left(r^2 + \frac{300^2}{\pi^2 r^4}\right)}$

$A = \sqrt{\pi^2 r^4 + \frac{300^2}{r^2}}$

$$A' = \frac{1}{2}\left(\pi^2 r^4 + \frac{300^2}{r^2}\right)^{-\frac{1}{2}}\left[\pi^2 4r^3 - \frac{2(300^2)}{r^3}\right]$$

$$A' = \frac{4\pi^2 r^3 - 2(300^2)r^{-3}}{2\sqrt{\pi^2 r^4 + \left(\frac{300^2}{r^2}\right)}}$$

denominator never vanishes

$$0 = A'$$

$$\therefore \quad 0 = 4\pi^2 r^3 - \frac{2\,300^2}{r^3}$$

$$0 = 4\pi^2 r^6 - 2\,300^2$$

$$2\,300^2 = 4\pi^2 r^6$$

$$r^6 = \frac{2\cdot 300^2}{4\pi^2}$$

$$r = \sqrt[6]{\frac{2\cdot 300^2}{4\pi^2}} \doteq 4.07 \text{ cm}$$

Lets verify that the solution is a minimum

$$A'' \overset{\text{details skipped}}{=} \cdots = \frac{2(\pi^4 r^{12} + 585000\pi^2 r^6 + 8100000000)}{(\pi^2 r^6 + 90000)r^2(\pi^2 r^6 + 90000)}$$

$A''(4.07) > 0 \quad \therefore$ a minimum by the 2nd derivative test.

$$h = \frac{300}{\pi r^2} = \frac{300}{\pi(4.07)^2} = 5.76 \text{ cm}$$

$\therefore$ $r = 4.07$ cm
$h = 5.76$ cm