

Name: _____
Student ID: _____

Test 1

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Evaluate the following limits:

a. (2 marks)

$$\lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x + 1}$$

b. (3 marks)

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$$

c. (3 marks)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{81x^2 + 11}}{3x}$$

Question 2. (5 marks) Use the limit definition of the derivative to find the derivative of the function $f(x) = \frac{x}{x+3}$.

Question 3. (3 marks) State the conditions for a function, $f(x)$, to be continuous at $x = a$.

Question 4. (2 marks) State where the following function is not continuous:

$$f(x) = \frac{x^2 + 1}{x^3 - x}$$

Question 5. The distance travelled by a particle in meters per second is $s(t) = 1000 + 100t - 10t^2 + t^3$.

- a. (1 mark) Find the function that describes the velocity of the particle.
- b. (1 mark) Find the function that describes the acceleration of the particle.
- c. (1 mark) What is the velocity and acceleration of the particle at $t = 3$.
- d. (1 mark) What can be said about the particle when the acceleration and velocity have different signs.

Question 6. Find the derivative of the following functions:

a. (3 marks)

$$h(t) = \frac{t^2 + t^{3/2} + \sqrt{t} + 1}{\sqrt{t}}$$

b. (3 marks)

$$f(z) = (z^5 + z^3 + z)(z^8 + z^6 + z + 1)$$

c. (4 marks)

$$g(t) = \frac{4t^3 + 5t}{2\sqrt{t} + 3t}$$

d. (5 marks)

$$y(x) = \left(\frac{7x^2 + 1}{x^3 + 2} \right)^7$$

Question 7. (5 marks) Find the equation of the tangent to the curve $y = x\sqrt{2x+1}$ at $x = 4$.

Question 8. (5 marks) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the relation

$$xy = y^3 + 2$$

Bonus. (4 marks) Find the derivative of the following function(*do not simplify*).

$$f(x) = \left[\left(\frac{x^2 + 1}{x^4 + x} \right) \left(\frac{\sqrt{x}}{x^3 + 1} \right) \right]^{101} \sqrt{\frac{x^{3/2} + x}{x + 1}}$$