Dawson College: Calculus I: 201-NYA-05-S06: Winter 2009

Name: YANN LAMONTAGNE
Student ID:

Test 1

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Evaluate the following limits:

a. *(2 marks)*

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1}$$

b. *(3 marks)*

$$\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x-4}$$

c. *(3 marks)*

$$\lim_{x \to -\infty} \frac{\sqrt{81x^2 + 11}}{3x}$$

a) $\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1}$ has IF $\frac{0}{0}$

$= \lim_{x \to -1} \frac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}}$

$= \lim_{x \to -1} 2x - 3 = 2(-1) - 3 = -5$

b) $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x-4}$ has IF $\frac{0}{0}$

$= \lim_{x \to 4} \frac{(\sqrt{x+5} - 3)}{(x-4)} \frac{(\sqrt{x+5} + 3)}{(\sqrt{x+5} + 3)}$

$= \lim_{x \to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \lim_{x \to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x+5}+3)}$

$= \lim_{x \to 4} \frac{1}{\sqrt{x+5}+3}$

$= \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$

c) $\lim_{x \to -\infty} \frac{\sqrt{81x^2+11}}{3x} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$

$= \lim_{x \to -\infty} \frac{\sqrt{81x^2+11 \left(\frac{1}{x^2}\right)}}{3x\left(\frac{1}{x}\right)}$

$= \lim_{x \to -\infty} \frac{\sqrt{\frac{81\cancel{x^2}}{\cancel{x^2}} + \frac{11}{x^2}}}{\frac{3\cancel{x}}{\cancel{x}}}$

$= \lim_{x \to -\infty} \frac{\sqrt{81 + \cancel{\frac{11}{x^2}}^{\,0}}}{3}$

$= \frac{\sqrt{81}}{3} = \frac{9}{3} = 3$

Question 2. *(5 marks)* Use the limit definition of the derivative to find the derivative of the function $f(x) = \frac{x}{x+3}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h+3} - \frac{x}{x+3}}{h} \qquad LCD = (x+3)(x+h+3)$$

$$= \lim_{x \to 0} \frac{\frac{(x+h)(x+3) - x(x+h+3)}{(x+h+3)(x+3)}}{h}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2} + \cancel{xh} + \cancel{3x} + 3h - \cancel{x^2} - \cancel{xh} - \cancel{3x}}{h(x+h+3)(x+3)}$$

$$= \lim_{x \to 0} \frac{3\cancel{h}}{\cancel{h}(x+h+3)(x+3)}$$

$$= \frac{3}{(x+3)(x+3)} = \frac{3}{(x+3)^2}$$

Question 3. *(3 marks)* State the conditions for a function, $f(x)$, to be continuous at $x = a$.

1. $f(a)$ must be defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Question 4. *(2 marks)* State where the following function is not continuous:

$$f(x) = \frac{x^2+1}{x^3-x}$$

a rational function is continuous everywhere except where the denominator vanishes.

$x^3 - x = 0$

$x(x^2-1) = 0$

$x(x-1)(x+1) = 0$

$x = 0$ $\quad x-1=0$ $\quad x+1 = 0$

$x = 1$ $\quad x = -1$

$\therefore$ not continuous at $x = 0, 1, -1$.

Question 5. The distance travelled by a particle in meters per second is $s(t) = 1000 + 100t - 10t^2 + t^3$.

a. *(1 mark)* Find the function that describes the velocity of the particle.

b. *(1 mark)* Find the function that describes the acceleration of the particle.

c. *(1 mark)* What is the velocity and acceleration of the particle at $t = 3$.

d. *(1 mark)* What can be said about the particle when the acceleration and velocity have different signs.

a) $v(t) = s'(t)$

$= 100 - 20t + 3t^2$

b) $a(t) = v'(t)$

$= -20 + 6t$

c) @ $t = 3$

$v(3) = 100 - 20(3) + 3(3)^2 = 67$

$a(3) = -20 + 6(3) = -2$

d) The velocity is diminishing.

Question 6. Find the derivative of the following functions:

a. *(3 marks)*

$$h(t) = \frac{t^2+t^{3/2}+\sqrt{t}+1}{\sqrt{t}} = \frac{t^2+t^{3/2}+t^{\frac{1}{2}}+1}{t^{\frac{1}{2}}} = t^{\frac{3}{2}}+t+1+t^{-\frac{1}{2}}$$

b. *(3 marks)*

$$f(z) = (z^5+z^3+z)(z^8+z^6+z+1)$$

$$h'(t) = \frac{3}{2}t^{\frac{1}{2}}+1-\frac{1}{2}t^{-\frac{3}{2}}$$

c. *(4 marks)*

$$g(t) = \frac{4t^3+5t}{2\sqrt{t}+3t}$$

b) $f'(z) = (5z^4+3z^2+1)(z^8+z^6+z+1)$

$+ (z^5+z^3+z)(8z^7+6z^5+1)$

d. *(5 marks)*

$$y(x) = \left(\frac{7x^2+1}{x^3+2}\right)^7$$

$= 5z^{12}+5z^{10}+5z^5+5z^4+3z^{10}+3z^8+3z^3+3z^2$

$+z^8+z^6+z+1+8z^{12}+6z^{10}+z^5+8z^{10}$

$+6z^8+8z^8+6z^6+z+z^3$

$= 13z^{12}+22z^{10}+18z^8+7z^6+6z^5+5z^4$

$+4z^3+2z+1+3z^2$

c) $$g'(t) = \frac{(12t^2+5)(2\sqrt{t}+3t)-(4t^3+5t)(t^{-\frac{1}{2}}+3)}{(2\sqrt{t}+3t)^2}$$

$$= \frac{24t^{\frac{5}{2}}+36t^3+10\sqrt{t}+15t-[4t^{\frac{5}{2}}+12t^3+5t^{\frac{1}{2}}+15t]}{(2\sqrt{t}+3t)^2}$$

$$= \frac{20t^{5/2}+24t^3+5\sqrt{t}}{(2\sqrt{t}+3t)^2}$$

d) $$y'(x) = 7\left(\frac{7x^2+1}{x^3+2}\right)^6\left[\frac{(14x)(x^3+2)-(7x^2+1)(3x^2)}{(x^3+2)^2}\right]$$

$$= \frac{7(7x^2+1)^6(-7x^4-3x^2+28x)}{(x^3+2)^8}$$

Question 7. *(5 marks)* Find the equation of the tangent to the curve $y = x\sqrt{2x+1}$ at $x = 4$.

$$y'(x) = \sqrt{2x+1} + x\left(\frac{1}{2}\right)(2x+1)^{-\frac{1}{2}} \cdot 2$$

$$= \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}}$$

$$= \frac{2x+1+x}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$$

$$m_{tan} = y'(4) = \frac{3(4)+1}{\sqrt{2(4)+1}} = \frac{13}{\sqrt{9}} = \frac{13}{3}$$

To find the eqn we need a point on the line

@ $x = 4$ $\quad y = 4\sqrt{2(4)+1} = 4\sqrt{9} = 4 \cdot 3 = 12$

$\therefore p\,(4, 12)$

Hence

$$y = mx + b$$

$$12 = \frac{13}{3}(4) + b$$

$$12 = \frac{52}{3} + b$$

$$b = \frac{36-52}{3} = \frac{-16}{3}$$

$$\therefore \; y = \frac{13}{3}x - \frac{16}{3}$$

Question 8. *(5 marks)* Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the relation

$$xy = y^3 + 2$$

$$\frac{d}{dx}[xy] = \frac{d}{dx}[y^3 + 2]$$

$$3y^2y' = y + xy'$$

$$3y^2y' - xy' = y$$

$$y'(3y^2 - x) = y$$

$$y' = \frac{y}{3y^2 - x}$$

$$\frac{d}{dx}[y'] = \frac{d}{dx}\left[\frac{y}{3y^2 - x}\right]$$

$$y'' = \frac{y'(3y^2 - x) - y(6yy' - 1)}{(3y^2 - x)^2}$$

$$= \frac{\frac{y}{3y^2-x}(3y^2 - x) - y\left(6y\left(\frac{y}{3y^2-x}\right) - 1\right)}{(3y^2 - x)^2}$$

$$= \frac{y(3y^2 - x) - y(6y^2 - (3y^2 - x))}{(3y^2 - x)^3}$$

$$= \frac{3y^3 - xy - 6y^3 + 3y^3 - xy}{(3y^2 - x)^3}$$

$$= \frac{-2xy}{(3y^2 - x)^3}$$

Bonus. *(4 marks)* Find the derivative of the following function*(do not simplify)*.

$$f(x) = \left[\left(\frac{x^2+1}{x^4+x}\right)\left(\frac{\sqrt{x}}{x^3+1}\right)\right]^{101}\sqrt{\frac{x^{3/2}+x}{x+1}} = \left[\frac{x^{5/2} + x^{\frac{1}{2}}}{x^7 + 2x^4 + x}\right]^{101}\sqrt{\frac{x^{3/2}+x}{x+1}}$$

$$f'(x) = 101\left[\frac{x^{5/2} + x^{\frac{1}{2}}}{x^7 + 2x^4 + x}\right]^{100}\left[\frac{\left(\frac{5}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)(x^7 + 2x^4 + x) - (x^{\frac{5}{2}} + x^{\frac{1}{2}})(7x^6 + 8x^3 + 1)}{(x^7 + 2x^4 + x)^2}\right]$$

$$\times\sqrt{\frac{x^{3/2}+x}{x+1}} + \left[\frac{x^{5/2} + x^{\frac{1}{2}}}{x^7 + 2x^4 + x}\right]^{101}\frac{1}{2}\cdot\left[\frac{x^{3/2}+x}{x+1}\right]^{-\frac{1}{2}}\left[\frac{\left(\frac{3}{2}x^{\frac{1}{2}} + 1\right)(x+1) - (x^{3/2} + x}{(x+1)^2}\right.$$