**Dawson College**: **Calculus I**: 201-NYA-05-S06: Winter 2009

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# Test 2

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** Compute the derivative of the following functions *(1 mark each)*:

a.

$f(x) = \sin x$ $\quad f'(x) = \cos x$

b.

$f(x) = \cos x$ $\quad f'(x) = -\sin x$

c.

$f(x) = \csc x$ $\quad f'(x) = -\csc x \cot x$

d.

$f(x) = \sec x$ $\quad f'(x) = \sec x \tan x$

e.

$f(x) = \tan x$ $\quad f'(x) = \sec^2 x$

f.

$f(x) = \cot x$ $\quad f'(x) = -\csc^2 x$

g.

$f(x) = \arcsin x$ $\quad f'(x) = \frac{1}{\sqrt{1-x^2}}$

h.

$f(x) = \arccos x$ $\quad f'(x) = \frac{-1}{\sqrt{1-x^2}}$

i.

$f(x) = \arctan x$ $\quad f'(x) = \frac{1}{1+x^2}$

j.

$f(x) = \ln x$ $\quad f'(x) = \frac{1}{x}$

k.

$f(x) = e^x$ $\quad f'(x) = e^x$

**Question 2.** Compute the derivative of the following functions. *(Do not simplify.)*

a. *(3 marks)*

$$g(t) = te^{\arctan 5t}$$

b. *(3 marks)*

$$z(x) = (\csc 4x)(\tan 3x)$$

c. *(3 marks)*

$$h(z) = \ln\left[(z+2)^2(3z+1)^3\right]$$

a) $g'(t) = e^{\arctan 5t} + te^{\arctan 5t} \cdot \left(\frac{1}{1+(5t)^2}\right) \cdot 5$

b) $z'(x) = -\csc 4x \cot 4x \cdot 4 \cdot \tan 3x + \csc 4x \cdot \sec^2 3x \cdot 3$

c) $h(z) = \ln(z+2)^2 + \ln(3z+1)^3$

$= 2\ln(z+2) + 3\ln(3z+1)$

$h'(z) = \frac{2}{z+2} + \frac{3}{3z+1} \cdot 3$

**Question 3.** Compute the derivative of the following functions. *(Do not simplify.)*

a. *(3 marks)*

$$f(x) = \frac{e^{\pi x}+1}{\cos \pi x}$$

b. *(3 marks)*

$$g(z) = \sqrt{\arcsin(\ln(\sin 3x))}$$

c. *(3 marks)*

$$s(t) = ((\sec x)\ln(\cos x))^7$$

a) $f'(x) = \dfrac{e^{\pi x} \cdot \pi \cos \pi x - (e^{\pi x}+1) - \sin \pi x}{(\cos \pi x)^2}$

b) $g'(z) = \frac{1}{2}(\arcsin(\ln(\sin 3x)))^{-\frac{1}{2}} \cdot \dfrac{1}{\sqrt{1-(\ln \sin 3x)^2}} \cdot \dfrac{1}{\sin 3x} \cdot \cos 3x \cdot 3$

c) $s'(t) = 7(\sec x \ln \cos x)^6 \left[\sec x \tan x \ln \cos x + \sec x \left(\frac{1}{\cos x}\right) - \sin x\right]$

**Question 4.** *(5 marks)* Find the equation of the normal to the curve $y = \arctan(\ln x)$ at $x = 1$.

Lets find the slope of the tangent at $x=1$

$$y'(x) = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

$$m_{tan} = y'(1) = \frac{1}{1+\underbrace{(\ln 1)^2}_{0}} \cdot \frac{1}{1} = 1 \quad \therefore m_{normal} = -1$$

Lets find the point on the curve at $x=1$

$$y = \arctan(\ln 1)$$
$$= \arctan 0$$
$$= 0$$

$\therefore$ point on curve and normal $(1,0)$

$$y = m_{normal}\, x + b$$

$$0 = -1(1) + b$$

$$1 = b$$

$$\therefore \quad y = -x + 1$$

**Question 5.** *(4 marks)* Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the relation

$\ln(xy) = x^2$

$$x^2 = \ln x + \ln y$$

$$\frac{d}{dx}\left[x^2\right] = \frac{d}{dx}\left[\ln x + \ln y\right]$$

$$2x = \frac{1}{x} + \frac{1}{y}y'$$

$$\frac{y'}{y} = 2x - \frac{1}{x}$$

$$\frac{y'}{y} = \frac{2x^2-1}{x}$$

$$y' = \frac{(2x^2-1)y}{x}$$

$$\frac{d}{dx}\left[y'\right] = \frac{d}{dx}\left[\frac{(2x^2-1)y}{x}\right]$$

$$y'' = \frac{x\left[4xy + (2x^2-1)y'\right] - (2x^2-1)y}{x^2}$$

$$y'' = \frac{x\left[4xy + (2x^2-1)\frac{(2x^2-1)y}{x}\right] - (2x^2-1)y}{x^2}$$

$$= \frac{4x^2y + (2x^2-1)^2y - (2x^2-1)y}{x^2}$$

$$= \frac{4x^2y + 4x^4y - 4x^2y + y - 2x^2y + y}{x^2}$$

$$= \frac{4x^4y - 2x^2y + 2y}{x^2}$$

**Question 6.** *(5 marks)* Find a root of $x^3 + 2x^2 - x - 1 = 0$ to at least four decimal places by using Newton's Method and let $x_0 = 0.8$.

Let $f(x) = x^3 + 2x^2 - x - 1$

$\Rightarrow f'(x) = 3x^2 + 4x - 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ ← Newton's Method

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $f(x_n)/f'(x_n)$ |
|---|---|---|---|---|
| 0 | 0.8 | −0.008 | 4.12 | −0.001941747573 |
| 1 | 0.8019417476 | 0.0000016597 | 4.137098690 | 0.0000040117486 |
| 2 | $\boxed{0.80193}77359$ | $1 \times 10^{-9}$ | 4.137063341 | $2.4 \times 10^{-10}$ |
| 3 | $\boxed{0.80193}87931$ | | | |

∴ after 3 iterations we get a approx. root to five decimal places

**Bonus.** *(3 marks)* Find the derivative of $y = \operatorname{arcsec} x$ *(hint: use implicit differentiation).*

$\sec y = \sec \operatorname{arcsec} x$

$\sec y = x$

Then by implicit diff.

$\frac{d}{dx}\left[\sec y\right] = \frac{d}{dx}[x]$

$\sec y \tan y \, y' = 1$

$y' = \frac{1}{\sec y \tan y}$

$1 + \tan^2 y = \sec^2 y$

$* \quad \tan y = \sqrt{\sec^2 y - 1}$

$y' = \frac{1}{x \tan y}$ since $x = \sec y$

$y' = \frac{1}{x\sqrt{\sec^2 y - 1}}$ by $*$

$y' = \frac{1}{x\sqrt{x^2 - 1}}$ since $x = \sec y$