

Test 3

This test is graded out of 41 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Compute the indefinite integral(1 mark each):

a.

$$\int \sec x \, dx$$

b.

$$\int \csc x \, dx$$

c.

$$\int \tan x \, dx$$

d.

$$\int \cot x \, dx$$

e.

$$\int e^x \, dx$$

f.

$$\int \frac{1}{x} \, dx$$

g. (bonus)

$$\int \frac{1}{1+x^2} \, dx$$

h. (bonus)

$$\int \frac{1}{\sqrt{1-x^2}} \, dx$$

Question 2. Compute the indefinite integral.

a. (3 marks)

$$\int \frac{z(z-1)}{\sqrt{z}} dz$$

b. (5 marks)

$$\int \sin \pi x \cos^7 \pi x dx$$

c. (4 marks)

$$\int \theta e^{\theta^2} d\theta$$

d. (2 bonus marks)

$$\int \frac{1}{e^x + e^{-x}} dx$$

Question 3. (5 marks) A ladder is slipping down along a vertical wall. If the ladder is 4 m long and the top of it is slipping at the constant rate of 3 m/s, how fast is the bottom of the ladder moving along the ground when the bottom is 2 m from the wall?

Question 4. Let $f(x) = x(x - 4)^3$

- a. (2 marks) Find the x and y intercepts of $f(x)$.
- b. (2 marks) Find $f'(x)$ and solve for the critical points.
- c. (2 marks) On what intervals is $f(x)$ increasing/decreasing?
- d. (1 mark) Identify the relative minimum and maximum.
- e. (1 mark) Find $f''(x)$.
- f. (2 marks) On what intervals is $f(x)$ concave up/down?
- g. (1 mark) Identify any inflection points.
- h. (2 marks) Sketch the graph of $f(x)$.

Question 5. (5 marks) A rectangular building covering $7\,000\text{ m}^2$ is to be built on a rectangular lot. If the building is to be 10 m from the boundary lot on each side and 20 m from the boundary in front and back, find the dimensions of the building if the area of the lot is a minimum.

Bonus. (3 marks) A light in a garage is 3 m above the floor and 4 m behind the door. If the garage door descends vertically at 0.5 m/s, how fast is the door's shadow moving toward the garage when the door is $\frac{\sqrt{3}}{2}$ above the floor.