Test 4

This test is graded out of 32 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Compute the definite integral:

a. (4 marks)

$$\int_0^{1/\sqrt{2}} \frac{e^{\arcsin x}}{\sqrt{1-x^2}} \, dx$$

b. (4 marks)

$$\int_0^1 \frac{x^2 + x^3}{4x^3 + 3x^4 + 303} \, dx$$

Question 2. (5 marks) Find the area of the region bounded by the graphs of $y = x^2 + 5x$ and $y = 3 - x^2$.

Question 3.

a. (5 marks) Use the Trapezoidal Rule with n = 4 to approximate the area under the curve

$$f(x) = x\sqrt{x^2 + 1}$$

on [0,1].

b. (4 marks) Find the exact solution and compare with the approximate solution.

Question 4. (5 marks) Using the disk method, find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = x^{2/3}$, y = 0, x = 0 and x = 1 about the *x*-axis.

Question 5. (5 marks) Using the shell method, find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = x^3$, y = x, x = 0 and x = 1 about the *y*-axis.

Bonus. (1 mark each) Integrate the following indefinite integrals.

a.

$$\int \frac{1}{x^2 + 4x + 5} \, dx$$

b.

$$\int \frac{\sqrt{x}-1}{\sqrt{x}+1} \, dx$$

c.

$$\int \frac{\cos^5 x}{\sqrt{\sin x}} \, dx$$