Dawson College: **Functions and Trigonometry**: 201-009-50-S02: Winter 2009

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Test 1

This test is graded out of 49 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Simplify, expressing the answers with positive exponents only:

a. *(4 marks)*

$$\left[\frac{-2x^2y^{-1}}{(-3x^{-2}y^4)^2}\right]^{-3} = \frac{(-2)^{-3}x^{-6}y^{3}}{(-3x^{-2}y^4)^{-6}} = \frac{y^3(-3x^{-2}y^4)^6}{(-2)^3x^6}$$

$$= \frac{y^3(-3)^6x^{-12}y^{24}}{-8x^6}$$

$$= \frac{-729y^{27}}{8x^{18}}$$

b. *(4 marks)*

$$\sqrt[3]{\frac{-27x^2y^{-2}z^0}{\sqrt{64z^6y^4}}}$$

$$= \left(\frac{-27x^2y^{-2}}{(64z^6y^4)^{\frac{1}{2}}}\right)^{1/3}$$

$$= \frac{(-27)^{\frac{1}{3}}x^{2/3}y^{-2/3}}{\left((64)^{\frac{1}{2}}z^3y^2\right)^{1/3}}$$

$$= \frac{-3x^{2/3}}{(8z^3y^2)^{1/3}y^{2/3}}$$

$$= \frac{-3x^{2/3}}{2z\,y^{4/3}}$$

Question 2. Expand and simplify:

a. *(2 marks)*

$x^3(2x-3)^2$ $= x^3(2x-3)(2x-3) = x^3(4x^2-12x+9)$

$= 4x^5 - 12x^4 + 9x^3$

b. *(2 marks)*

$(x-1)^3$ $= (x-1)(x-1)(x-1) = [x^2-2x+1][x-1]$

$= x^3 - 3x^2 + 3x - 1$

c. *(2 marks)*

$(2\sqrt{3}-3\sqrt{5})(4\sqrt{5}+\sqrt{7})$

$= 8\sqrt{15} + 2\sqrt{21} - 12\cdot 5 - 3\sqrt{35}$

$= 8\sqrt{15} + 2\sqrt{21} - 60 - 3\sqrt{35}$

Question 3. *(4 marks)* Divide by long division.

$\frac{2x^3+5x^2-1}{x-2}$

$$\begin{array}{r}
2x^2 + 9x + 18 \\
x-2 \,\big)\, 2x^3 + 5x^2 + 0x - 1 \\
-(2x^3 - 4x^2) \\
\hline
9x^2 + 0x \\
-(9x^2 - 18x) \\
\hline
18x - 1 \\
-(18x - 36) \\
\hline
35
\end{array}$$

$\therefore \frac{2x^3+5x^2-1}{x-2} = 2x^2+9x+18 + \frac{35}{x-2}$

Question 4. Factor completely:

a. *(1 mark)*

$x^2 - 9x + 20 = (x-4)(x-5)$

b. *(1 mark)*

$9x^2 - 16 = (3x-4)(3x+4)$

c. *(2 marks)*

$8x^2 + 14x + 5 = 8x^2 + 14x + 5$

d. *(2 marks)*

$15x^4 - 25x^3 + 10x^2$

$8x^2(5) = 40x^2 = ab$

s.t. $a+b = 14x$

$4x + 10x = 14x$

$= 8x^2 + 4x + 10x + 5$

$= 4x(2x+1) + 5(2x+1)$

$= (4x+5)(2x+1)$

d) $15x^4 - 25x^3 + 10x^2$

$= 5x^2(3x^2 - 5x + 2)$

$= 5x^2(3x-2)(x-1)$

Question 5. Simplify the rational expressions:

a. *(5 marks)*

$$\frac{a^2-ax}{3ax-2x^2}\times\frac{4ax+2x^2}{ax-x^2}\div\frac{4a^2+2ax}{9a-6x} \quad = \frac{a(a-x)}{x(\cancel{3a-2x})}\times\frac{\cancel{2x}(\cancel{2a+x})}{\cancel{x}(a-x)} \cdot \frac{3(\cancel{3a-2x})}{2a(\cancel{2a+x})}$$

b. *(5 marks)*

$$\frac{2-5x}{x+3}-\frac{3+x}{3-x}+\frac{2x(2x-11)}{x^2-9} \quad = \frac{3}{x}$$

$$= \frac{2-5x}{x+3} - \frac{3+x}{-(x-3)} + \frac{2x(2x-11)}{(x-3)(x+3)} \qquad LCD = (x+3)(x-3)$$

$$= \frac{(2-5x)(x-3)}{(x+3)(x-3)} + \frac{(x+3)(x+3)}{(x-3)(x+3)} + \frac{2x(2x-11)}{(x-3)(x+3)}$$

$$= \frac{2x-6-\cancel{5x^2}+15x + \cancel{x^2}+6x+9+\cancel{4x^2}-22x}{(x+3)(x-3)}$$

$$= \frac{\cancel{x+3}}{\cancel{(x+3)}(x-3)}$$

$$= \frac{1}{x-3}$$

Question 6. *(5 marks)* Simplify the complex fractions:

$$\frac{\frac{2}{1-x^2}}{\frac{1}{1-x}-\frac{1}{1+x}} = \frac{\frac{2}{1-x^2}}{\frac{1+x}{(1-x)(1+x)}-\frac{1-x}{(1+x)(1-x)}}$$

LCD for denom. $= (1-x)(1+x)$

$$= \frac{\frac{2}{1-x^2}}{\frac{2x}{(1-x)(1+x)}}$$

$$= \frac{2(1-x)(1+x)}{2x(1-x^2)} = \frac{\cancel{(1-x)}\cancel{(1+x)}}{x\cancel{(1-x)}\cancel{(1+x)}} = \frac{1}{x}$$

Question 7. Rationalize the denominator and simplify:

a. *(1 mark)*

$$\frac{2}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{2\sqrt{5}}{5}$$

b. *(2 marks)*

$$\frac{7}{1+\sqrt{7}}\left(\frac{1-\sqrt{7}}{1-\sqrt{7}}\right) = \frac{7-7\sqrt{7}}{1-7}$$

$$= \frac{7-7\sqrt{7}}{-6}$$

Question 7. *(3 marks)* Find the number such that 7 less than 4 times itself is 13.

$$4x - 7 = 13$$
$$4x = 20$$
$$x = 5$$

Question 8. *(4 marks)* Solve for x:

$$(x^2 - 25)(3x^2 + 4x - 6) = 0$$

$$x^2 - 25 = 0$$
$$x^2 = 25$$
$$x = \pm\sqrt{25}$$
$$x = \pm 5$$

$$3x^2 + 4x - 6 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(3)(-6)}}{2(3)}$$
$$= \frac{-4 \pm \sqrt{16 + 72}}{6}$$
$$= \frac{-4 \pm \sqrt{88}}{6}$$
$$= \frac{-4 \pm \sqrt{4}\sqrt{22}}{6}$$
$$= \frac{-4 \pm 2\sqrt{22}}{6}$$
$$= \frac{-2 \pm \sqrt{22}}{3}$$

$$\therefore x = \pm 5, \frac{-2 \pm \sqrt{22}}{3}$$

Bonus.

a. *(2 marks)* The *rational root theorem* states that if the polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ has coefficients a_i that are all integers and $p(x)$ has a rational root r, then a_0 is divisible by r. Let $p(x) = x^3 + 4x^2 + x - 6$ then state the possible rational roots.

b. *(2 marks)* The *factor theorem* states that if $p(r) = 0$ then $(x - r)$ is a factor of $p(x)$. Using the rational root theorem find all factors.

c. *(1 mark)* Factor $p(x)$.

a) $\pm 6, \pm 2, \pm 3, \pm 1$

b) $p(6) = 6^3 + 4 \cdot 6^2 + 6 - 6 = 360$

$p(-6) = (-6)^3 + 4(-6)^2 - 6 - 6 = -84$

$p(2) = 2^3 + 4(2)^2 + 6 - 6 = 24$

$p(-2) = (-2)^3 + 4(-2)^2 - 2 - 6 = 0$ $\therefore$ -2 is a root hence, $(x+2)$ is a factor

$p(3) = 3^3 + 4 \cdot 3^2 + 3 - 6 = 280$

$p(-3) = (-3)^3 + 4 \cdot (-3)^2 - 3 - 6 = 0$ $\therefore$ -3 is a root hence, $(x+3)$ is a factor

$p(1) = 1^3 + 4(1)^2 + 1 - 6 = 0$ $\therefore$ 1 is a root hence, $(x-1)$ is a factor

$p(-1) = (-1)^3 + 4(-1)^2 - 1 - 6 = -4$

c) $p(x) = x^3 + 4x^2 + x - 6$

$= (x+2)(x+3)(x-1)$