Dawson College: **Functions and Trigonometry**: 201-009-50-S02: Winter 2009

Name: YANN LAMONTAGNE
Student ID: ________________

Test 2

This test is graded out of 42 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Solve for x:

a. *(3 marks)*

$$\frac{4x+1}{2}-\frac{2x+3}{3}=\frac{5x-1}{4}$$

b. *(6 marks)*

$$\frac{x+4}{x^2-3x+2}-\frac{5}{x^2-4x+3}-\frac{x-4}{x^2-5x+6}=0$$

a.) $\frac{4x+1}{2} - \frac{2x+3}{3} = \frac{5x-1}{4}$ LCD = 12

$\frac{\cancel{12}^{6}(4x+1)}{\cancel{2}} - \frac{\cancel{12}^{4}(2x+3)}{\cancel{3}} = \frac{\cancel{12}^{3}(5x-1)}{\cancel{4}}$

$24x+6 - (8x+12) = 15x-3$

$24x+6-8x-12 = 15x-3$

$x = 3$

b) $\frac{x+4}{x^2-3x+2} - \frac{5}{x^2-4x+3} - \frac{x-4}{x^2-5x+6} = 0$

LCD = $(x-1)(x-2)(x-3)$

$\frac{x+4}{(x-1)(x-2)} - \frac{5}{(x-1)(x-3)} - \frac{x-4}{(x-2)(x-3)} = 0$

$\frac{(x+4)\cancel{(x-1)}\cancel{(x-2)}(x-3)}{\cancel{(x-1)}\cancel{(x-2)}} - \frac{5\cancel{(x-1)}(x-2)\cancel{(x-3)}}{\cancel{(x-1)}\cancel{(x-3)}} - \frac{(x-4)(x-1)\cancel{(x-2)}\cancel{(x-3)}}{\cancel{(x-2)}\cancel{(x-3)}} = 0$

$\cancel{x^2}+x-12-\cancel{5x}+10-\cancel{x^2}+\cancel{5x}-4 = 0$

$x = 6$

check sol:

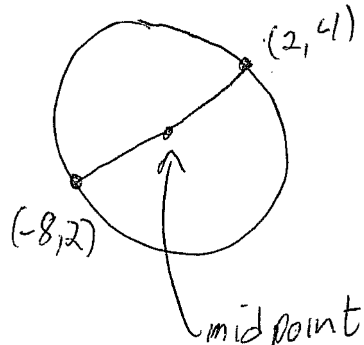
	$x=6$
$(x-1)(x-2)$	$\neq 0$
$(x-1)(x-3)$	$\neq 0$
$(x-2)(x-3)$	$\neq 0$

Question 2. *(4 marks)* State the equation of the circle given the two endpoints of a diameter: $(-8,2)$ and $(2,4)$.

$$(h,k) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-8+2}{2}, \frac{2+4}{2}\right)$$

$$= \left(\frac{-6}{2}, \frac{6}{2}\right) = (-3, 3)$$



$$r = \sqrt{(h-x_1)^2 + (k-y_2)^2}$$

$$= \sqrt{(-3-(-8))^2 + (3-2)^2}$$

$$= \sqrt{5^2 + 1^2}$$

$$= \sqrt{26}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-(-3))^2 + (y-3)^2 = (\sqrt{26})^2$$

$$(x+3)^2 + (y-3)^2 = 26$$

Question 3. *(2 marks)* State the domain and range of $f(x) = \sqrt{x-5}$.

Domain: $[5, \infty)$

$$x - 5 \geq 0$$
$$x \geq 5$$

Range: $[0, \infty)$

Question 8. Let $f(x) = -3x^2 + 12x - 3$.

a. *(2 marks)* Determine the vertex of $f(x)$.

b. *(1 mark)* Determine the orientation of the parabola and state whether the vertex is a minimum or maximum.

c. *(1 mark)* Determine the y-intercept.

d. *(1 mark)* Determine the x-intercept(s).

e. *(1 mark)* Sketch the graph of $f(x)$.

f. *(1 marks)* Determine the domain and range of $f(x)$.

b)  vertex a max.

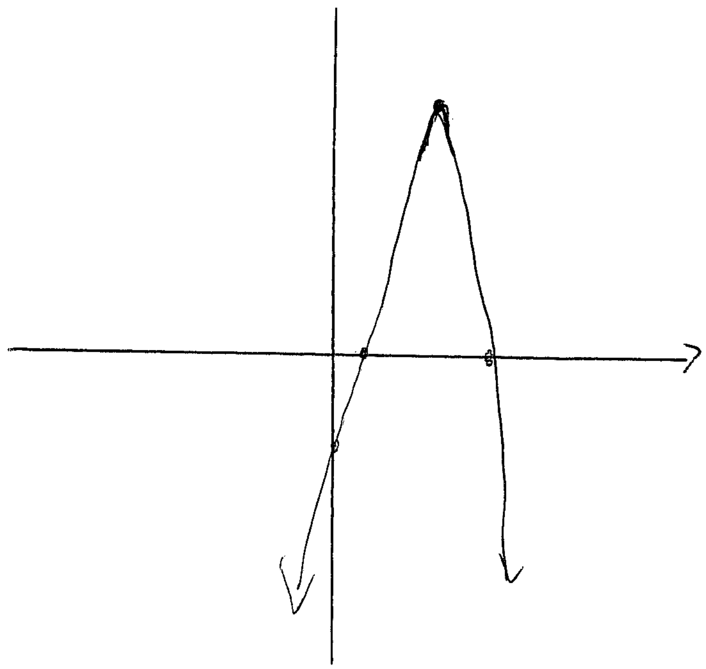
a)
$$\begin{aligned} f(x) &= -3[x^2 - 4x + 1] \\ &= -3[(x^2 - 4x + 4) - 4 + 1] \\ &= -3[(x-2)^2 - 3] \\ &= -3(x-2)^2 + 9 \end{aligned}$$

$\therefore$ vertex $(2, 9)$

c) y-int: $(0, -3)$

d)
$$\begin{aligned} 0 &= f(x) \\ 0 &= -3x^2 + 12x - 3 \\ 0 &= x^2 - 4x + 1 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2} \\ &= \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3} \\ &\doteq 3.73 \text{ and } 0.26 \end{aligned}$$

e)



f) Domain: $\mathbb{R}$

Range: $(-\infty, 9]$

Question 4. *(5 marks)* If $f(x) = \frac{x}{x+1}$ then find $\frac{f(x+h)-f(x)}{h}$.

$$\frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \frac{\frac{(x+h)(x+1)}{(x+h+1)(x+1)} - \frac{x(x+h+1)}{(x+1)(x+h+1)}}{h} \quad LCD = (x+h+1)(x+1)$$

$$= \frac{\frac{\cancel{x^2} + \cancel{xh} + \cancel{x} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{(x+h+1)(x+1)}}{h}$$

$$= \frac{\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \frac{1}{(x+h+1)(x+1)}$$

Question 5. If $f(x) = \sqrt{2x-3}$ and $g(x) = 7x^2 - 2$ then find:

a. *(1 mark)* $f\left(\frac{3}{2}\right)$

b. *(1 mark)* the value of x if $f(x) = 3$.

c. *(1 mark)* $5f(2) - 5g(2)$

a) $f\left(\frac{3}{2}\right) = \sqrt{\cancel{2}\left(\frac{3}{\cancel{2}}\right) - 3} = \sqrt{3-3} = \sqrt{0} = 0$

b) $3 = f(x)$

$3 = \sqrt{2x-3}$

$9 = 2x - 3$

$12 = 2x$

$6 = x$

c) $5f(2) - 5g(2)$

$= 5\sqrt{2(2)-3} - 5(7(2)^2 - 2)$

$= 5\sqrt{1} - 5(26)$

$= -125$

Question 6. *(4 marks)* Sketch a graph of the function $f(x) = -2x - 4$ using the x and y intercepts. *(clearly label the graph)*

x-int: let $f(x)=0$

$0 = f(x)$

$0 = -2x-4$

$4 = -2x$

$-2 = x$

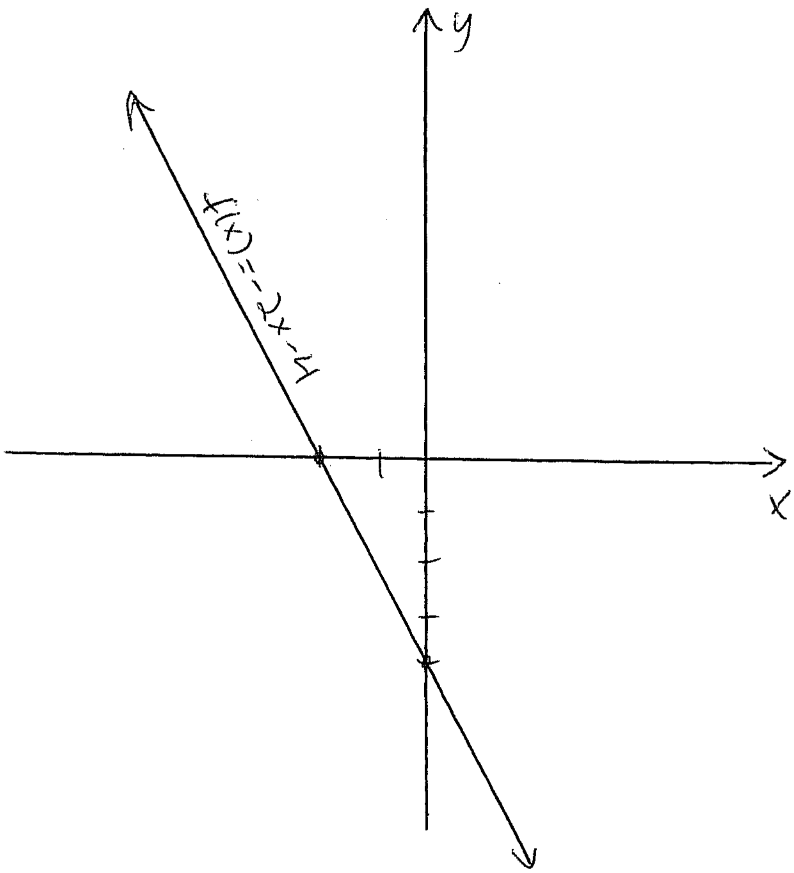
y-int: let $x=0$

$y = f(0) = -2(0)-4$

$y = -4$

$\therefore$ x-int $(-2,0)$

and y-int $(0,-4)$



Question 7. *(4 marks)* Find the equation of the line that passes through the points: $(-3,4)$ and $(7,-9)$.

$y = mx + b$

Slope:

$m = \frac{\Delta y}{\Delta x}$

$= \frac{-9-4}{7-(-3)}$

$= \frac{-13}{10}$

Solve for b:

$y = mx + b$

$y = \frac{-13}{10}x + b$ sub $(-3,4)$

$4 = \frac{-13}{10}(-3) + b$

$b = \frac{1}{10}$

$\therefore y = \frac{-13}{10}x + \frac{1}{10}$

Question 9. *(4 marks)* Find the equation of the line through $(4, \frac{1}{2})$ and perpendicular to $2y + 4x - 14 = 0$

$2y = -4x + 14$

$y = -2x + 7$

$\therefore m_{\perp} = \frac{1}{2}$

$y = m_{\perp} x + b$

$y = \frac{1}{2} x + b$ sub $(4, \frac{1}{2})$

$\frac{1}{2} = \frac{1}{2}(4) + b$

$\frac{1}{2} = 2 + b$

$b = -\frac{3}{2}$

$\therefore y = \frac{1}{2} x - \frac{3}{2}$

Bonus.

a. *(1 marks)* The *rational root theorem* states that if the polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has coefficients a_i that are all integers and $p(x)$ has a rational root r, then a_0 is divisible by r. Let $p(x) = x^3 + 4x^2 + x - 6$ then state the possible rational roots.

b. *(1 marks)* The *factor theorem* states that if $p(r) = 0$ then $(x - r)$ is a factor of $p(x)$. Using the rational root theorem find all factors of $p(x)$.

c. *(1 mark)* Factor $p(x)$.