Dawson College: **Functions and Trigonometry**: 201-009-50-S02: Winter 2009

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Test 3

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

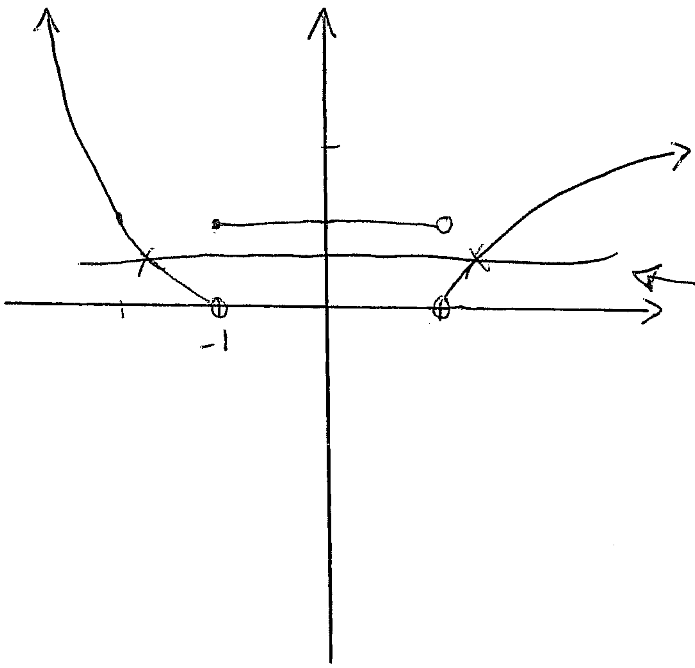
Question 1.

a. *(5 marks)* Sketch the graph of the piece-wise function:

$$f(x) = \begin{cases} (x+1)^2 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x < 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$$

b. *(2 marks)* State the domain and range of the above function, $f(x)$.

c. *(1 mark)* Is the above function, $f(x)$, injective, justify.



b) Domain of $f(x)$: $\mathbb{R} \setminus \{1\}$
Range of $f(x)$: $(0, \infty)$

c) Not injective, since fails the horizontal line test

Question 2.

a. *(4 marks)* Sketch the graph of the following function:

$$f(x) = 2^{x+1} - 2$$

b. *(4 marks)* Sketch the graph of the following function:

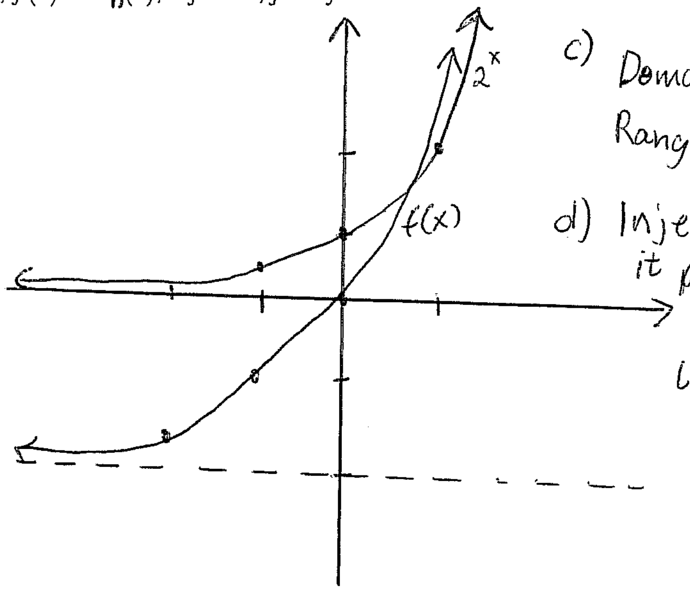
$$h(x) = \log_3(x+1) + 2$$

c. *(2 marks)* State the domain and range of the above two functions, $f(x)$ and $h(x)$.

d. *(2 marks)* Are the above two functions, $f(x)$ and $h(x)$, injective, justify.

a)

x	2^x
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$

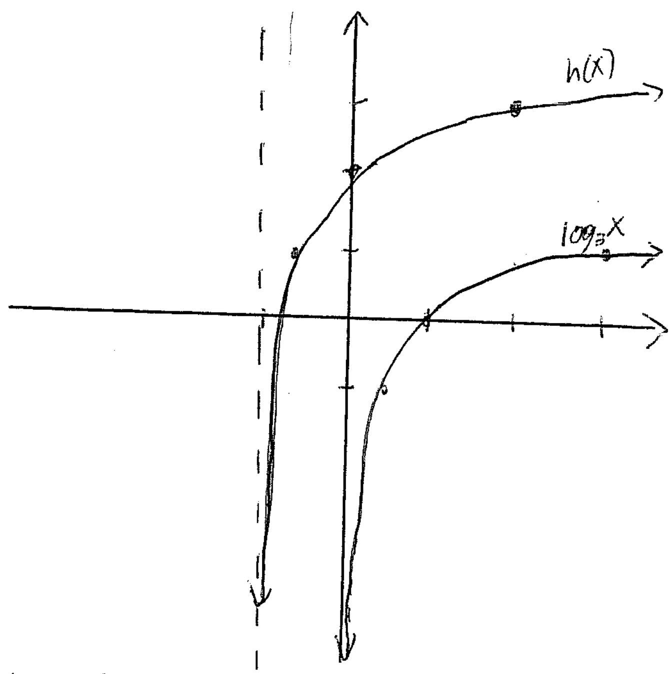


c) Domain: $\mathbb{R}$
Range: $(-2, \infty)$

d) Injective since it passes the horizontal line test.

b)

x	$\log_3 x$
$\frac{1}{3}$	$\log_3(\frac{1}{3}) = -1$
1	$\log_3(1) = 0$
3	$\log_3 3 = 1$



c) Domain: $(-1, \infty)$
Range: $\mathbb{R}$

d) Injective since it passes the horizontal line test.

Question 3.

a. *(1 mark)* Suppose $h(x)$ is injective and $h(2) = 3$, find $h^{-1}(3)$

b. *(4 marks)* Verify that $g(x)$ is the inverse of $f(x)$, where $g(x) = \sqrt[3]{x+1}$ and $f(x) = x^3 - 1$.

c. *(5 marks)* Find $k^{-1}(x)$ for the following function:

$$k(x) = \frac{2x+5}{3x-1}$$

a) $h^{-1}(3) = 2$

b) $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = x+1-1$
$= x$

$(g \circ f)(x) = g(f(x)) = g(x^3-1) = \sqrt[3]{x^3-1+1} = \sqrt[3]{x^3}$
$= x$

$\therefore$ $g(x)$ is the inverse of $f(x)$.

c) $k(x) = \dfrac{2x+5}{3x-1}$

$y = \dfrac{2x+5}{3x-1}$

$x = \dfrac{2y+5}{3y-1}$

$x(3y-1) = 2y+5$

$3xy - x = 2y+5$

$3xy - 2y = x+5$

$y(3x-2) = x+5$

$y = \dfrac{x+5}{3x-2}$

$\therefore$ $K^{-1}(x) = \dfrac{x+5}{3x-2}$

Question 4.

a. *(2 marks)* Express in terms of a simple logarithm:

$\log \sqrt[4]{xy^2\sqrt{z}}$ $= \log (xy^2\sqrt{z})^{\frac{1}{4}} = \frac{1}{4} \log (xy^2\sqrt{z}) = \frac{1}{4}[\log x + \log y^2 + \log z^{\frac{1}{2}}]$

$= \frac{1}{4}[\log x + 2\log y + \frac{1}{2}\log z]$

$= \frac{1}{4}\log x + \frac{1}{2}\log y + \frac{1}{8}\log z$

b. *(2 marks)* Express as a single logarithm with coefficient of one:

$\frac{1}{2}\log x + \frac{3}{2}\log 2y - \log x^2 y$ $= \log x^{\frac{1}{2}} + \log (2y)^{3/2} - \log x^2 y$

$= \log \frac{x^{\frac{1}{2}}(2y)^{3/2}}{x^2 y}$

$= \log \frac{\sqrt{8}\sqrt{y}}{x^{3/2}}$

c. *(5 marks)* Solve for x.

$\ln(x-2) + \ln(2x+3) = 2\ln x$

$\ln[(x-2)(2x+3)] = 2\ln x$

$\ln[2x^2 - x - 6] = \ln x^2$

$e^{\ln[2x^2 - x - 6]} = e^{\ln x^2}$

$2x^2 - x - 6 = x^2$ since e^x is the inverse of $\ln x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0$, $x = 3$

$x + 2 = 0$, $x = -2$ not valid since it make the inside of the log smaller than 0.

$\therefore x = 3$

Question 5. Let $f(x) = \log_2 x$ then

a. *(1 mark)* evaluate $f(0)$ if possible.

b. *(1 mark)* evaluate $f(1)$ if possible.

c. *(1 mark)* evaluate $f\left(\frac{1}{8}\right)$ if possible.

$f(0) =$ undefined

$f(1) = \log_2 1 = 0$

$f\left(\frac{1}{8}\right) = \log_2\left(\frac{1}{8}\right) = -3$

Question 6. Solve for x.

a. *(1 mark)*

$$\log_x 3 = -1 \iff x^{-1} = 3$$

$$\frac{1}{x} = 3$$

$$\frac{1}{3} = x$$

b. *(4 marks)*

$$4^{2x-1} = 7^{x+2}$$

$$\log_4 4^{2x-1} = \log_4 7^{x+2}$$

$$2x-1 = (x+2)\log_4 7$$

$$2x-1 = x\log_4 7 + 2\log_4 7$$

$$2x - x\log_4 7 = 1 + 2\log_4 7$$

$$x(2-\log_4 7) = 1 + 2\log_4 7$$

$$x = \frac{1+2\log_4 7}{2-\log_4 7}$$

Bonus.

a. *(2 marks)* Solve for x:

$$[\log_3(x-1)]^2 - 3\log_3(x-1) = 4$$

b. *(2 marks)* Solve for x:

$$2^x + 2^{-x} = 2$$

c. *(2 marks)* Solve for x:

$$\ln x - \log x = 1$$

let $u = \log_3(x-1)$ then

$u^2 - 3u = 4$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u - 4 = 0$

$u = 4$

$\log_3(x-1) = 4$

$x - 1 = 3^4$

$x - 1 = 81$

$x = 82$

$u + 1 = 0$

$u = -1$

$\log_3(x-1) = -1$

$x - 1 = 3^{-1}$

$x = \frac{1}{3} + 1$

$x = \frac{4}{3}$

$\therefore x = 82$ and $x = \frac{4}{3}$

b) $2^x + \frac{1}{2^x} = 2$

let $u = 2^x$

$u + \frac{1}{u} = 2$

$u^2 + 1 = 2u$

$u^2 - 2u + 1 = 0$

$(u-1)^2 = 0$

$u = 1$

$2^x = 1$

$\therefore x = 0$

c) $\ln x - \log x = 1$

$\frac{\log x}{\log e} - \log x = 1$

$\log x \left(\frac{1}{\log e} - 1\right) = 1$

$\log x = \frac{1}{\left(\frac{1}{\log e} - 1\right)}$

$x = 10^{\frac{1}{\left(\frac{1}{\log e} - 1\right)}}$