Dawson College: Functions and Trigonometry: 201-009-50-S02: Winter 2009

Name: _____

Student ID: _____

Test 4

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. What angle θ $(0° \leq \theta < 360°)$ is co-terminal to

a. *(2 marks)* 961°

b. *(2 marks)* $\frac{21\pi}{4}$

b) $\frac{21\pi}{4} = \theta + K \cdot 2\pi$

$K = \left\lfloor \frac{\frac{21\pi}{4}}{2\pi} \right\rfloor$

$\frac{21\pi}{4} = \theta + 2 \cdot 2\pi$

$= \lfloor 2.625 \rfloor$

$\theta = \frac{5\pi}{4}$

$= 2$

a) $961^\circ = \theta + K \cdot 360^\circ$

$961^\circ = \theta + 2 \cdot 360^\circ$

$\theta = 241^\circ$

$K = \left\lfloor \frac{961}{360} \right\rfloor$

$= \lfloor 2.6\overline{00} \rfloor$

$= 2$

Question 2. *(2 marks)* Consider an angle θ in standard position, find the quadrant in which its terminal edge lies, if $\sec\theta > 0$ and $\cot\theta < 0$

S $\cot\theta < 0$ | $\sec\theta > 0$ A

$\sec\theta > 0$, $\cot\theta < 0$

T C

$\therefore$ in the fourth quadrant

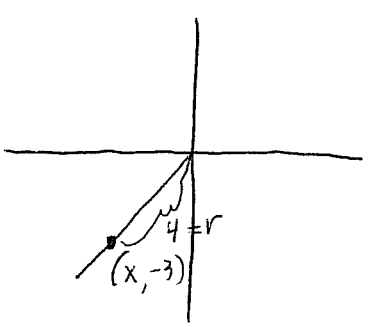
Question 3. *(4 marks)* Find the values of the 5 other trigonometric functions, if $\csc\theta = -\frac{4}{3}$ and $\tan\theta > 0$.

Sin | $\tan\theta > 0$ All

$\csc\theta < 0$, $\tan\theta > 0$ | $\csc\theta < 0$

Tan | Cos

$\csc\theta = \frac{r}{y} = \frac{4}{-3} \rightarrow \sin\theta = -\frac{3}{4}$



$r = \sqrt{x^2 + y^2}$

$4 = \sqrt{x^2 + (-3)^2}$

$16 = x^2 + 9$

$7 = x^2$

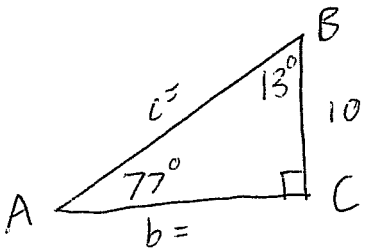
$\pm\sqrt{7} = x$

$-\sqrt{7} = x$

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{7}}{4} \rightarrow \sec\theta = \frac{-4}{\sqrt{7}}$

$\tan\theta = \frac{y}{x} = \frac{3}{\sqrt{7}} \rightarrow \cot\theta = \frac{\sqrt{7}}{3}$

Question 4. *(4 marks)* Solve the right triangle ABC ($C = 90°$) given: $a = 10, B = 13°$.



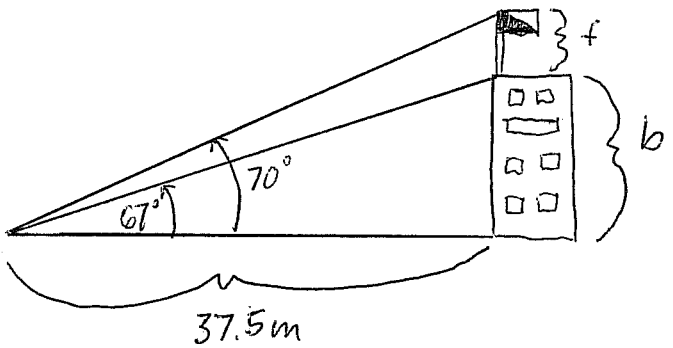
$$\tan 77^\circ = \frac{10}{b}$$

$$b = \frac{10}{\tan 77^\circ} \doteq 2.31$$

$$\sin 77^\circ = \frac{10}{c}$$

$$c = \frac{10}{\sin 77} \doteq 10.26$$

Question 5. *(4 marks)* A flagpole sits on the top of a building. From a point on the ground 37.5 m from the base of the building the angles of elevation to the top and bottom of the flagpole are 70° and 67° respectively. What is the height of the flagpole?



$$\tan 67^\circ = \frac{b}{37.5}$$

$$37.5 \tan 67^\circ = b$$

$$\tan 70^\circ = \frac{b+f}{37.5}$$

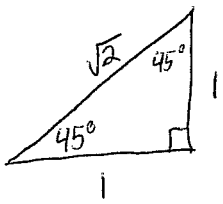
$$37.5 \tan 70^\circ = b + f$$

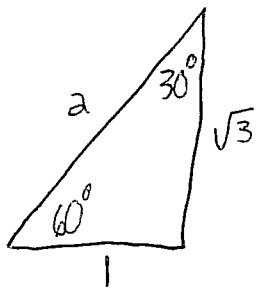
$$37.5 \tan 70^\circ - b = f$$

$$37.5 \tan 70 - 37.5 \tan 67 = f$$

$$14.7m \doteq f$$

Question 6. *(4 marks)* Draw the two "special triangles" which help identify the special angles. Label the angles of the triangles and the length of each side.





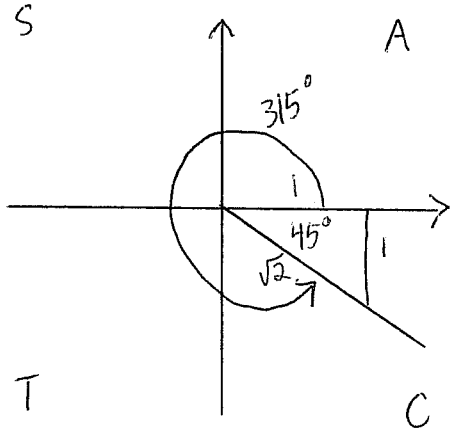
Question 7. Find the exact values of

a. *(4 marks)* csc 315°

b. *(4 marks)* sec $\frac{4\pi}{3}$

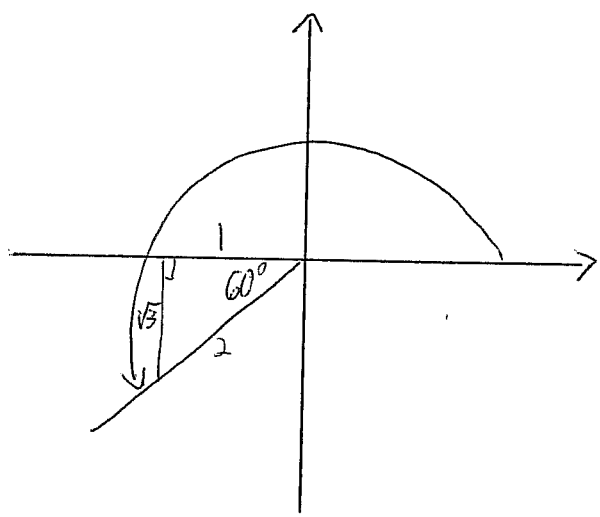
$\frac{4\pi}{3}\left(\frac{180}{\pi}\right)^{60} = 240°$

a)



$\csc 315° = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

b)



$\sec \frac{4\pi}{3} = \frac{r}{x}$ 1

$= \frac{2}{-1}$

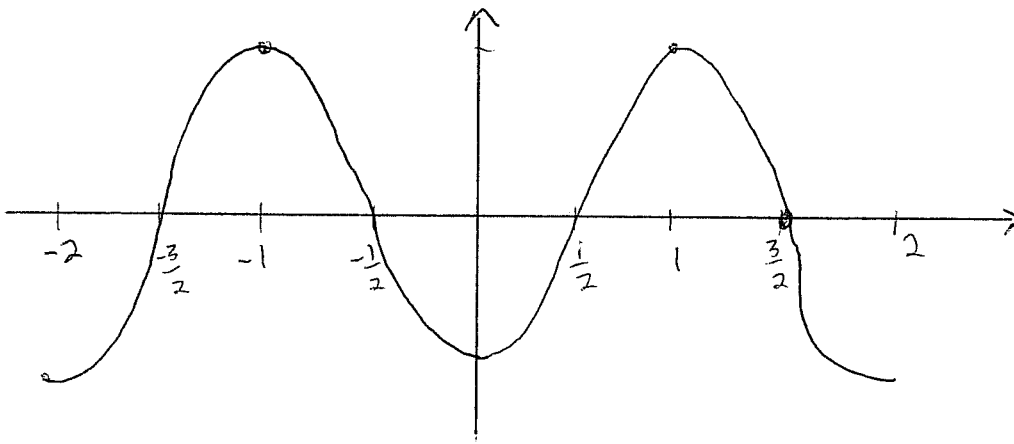
$= -2$

Question 8. *(4 marks)* Sketch the graph of the function $f(x) = -2\cos(\pi x)$ over the interval $[-2, 2]$.

amplitude $= |-2| = 2$

period $= \frac{2\pi}{\pi} = 2$

↑ flip w.r.t. x-axis



Question 10. *(4 marks)* Verify the following identity:

$$\frac{1-\cos 2x}{\sin 2x} \stackrel{?}{=} \tan x$$

$1 - \cos 2x \stackrel{?}{=} \tan x \sin 2x$

$1 - (1 - 2\sin^2 x) \stackrel{?}{=} \tan x \, 2 \sin x \cos x$

$1 - 1 + 2\sin^2 x \stackrel{?}{=} \frac{\sin x}{\cos x} (2) \sin x \cos x$

$2\sin^2 x = 2\sin^2 x$

Question 11. *(4 marks)* Solve for θ:

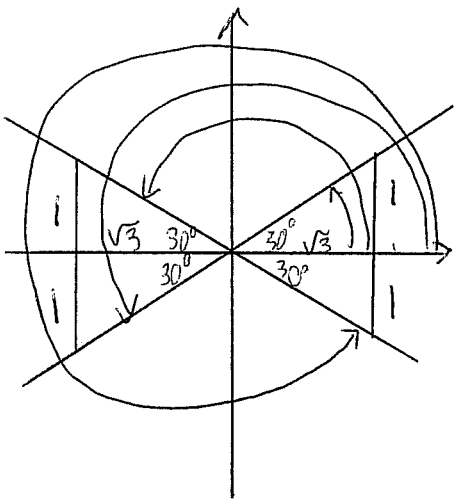
$$1 - 3\tan^2\theta = 0$$

$$1 = 3\tan^2\theta$$

$$\frac{1}{3} = \tan^2\theta$$

$$\sqrt{\frac{1}{3}} = \tan\theta$$

$$\frac{o}{a} = \pm\frac{1}{\sqrt{3}} = \tan\theta$$



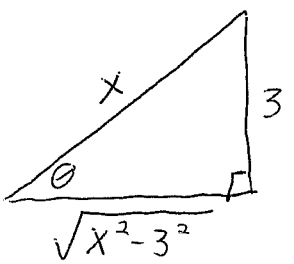
$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

Question 12. *(4 marks)* Find the value of:

$$\tan\left[\arcsin\left(\frac{3}{x}\right)\right]$$

$$\arcsin\frac{3}{x} = \theta$$

$$\frac{\text{opp.}}{\text{hyp.}} = \frac{3}{x} = \sin\theta$$



$$\tan\left(\arcsin\left(\frac{3}{x}\right)\right) = \tan(\theta)$$

$$= \frac{\text{opp.}}{\text{adj.}}$$

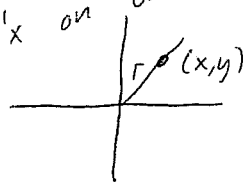
$$= \frac{3}{\sqrt{x^2-3^2}}$$

Bonus.

a. *(1 mark)* State the domain and range of $\csc x$.

b. *(2 marks)* Sketch the graph of $\csc x$.

c. *(2 marks)* Restrict $\csc x$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and sketch $\csc^{-1} x$ on that interval.

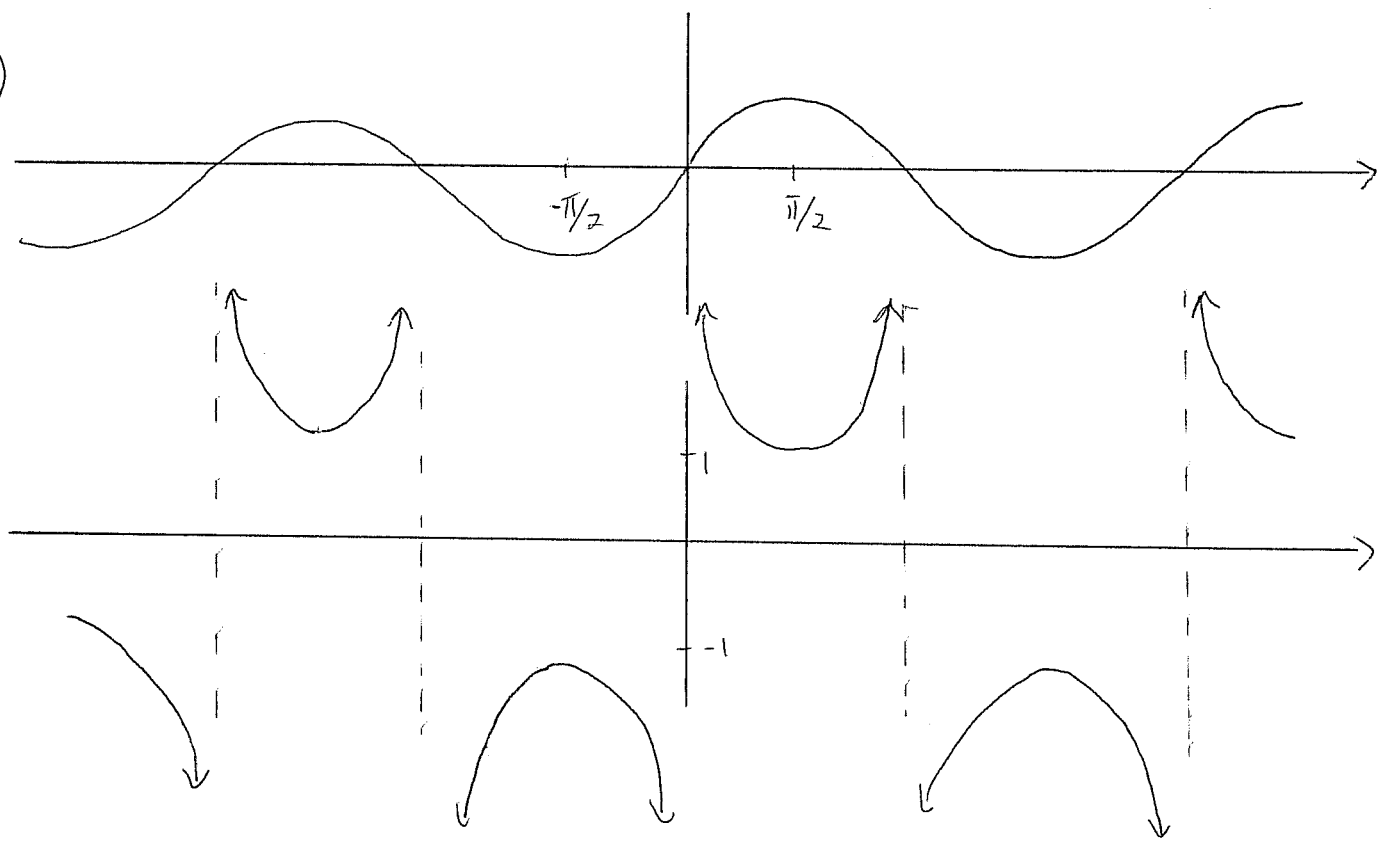


a) $\csc x = \frac{r}{y}$

Domain: $\mathbb{R} \setminus \pm 0^\circ, \pm 180^\circ, \pm 360^\circ, \ldots$

Range: $\mathbb{R} \setminus (-1, 1)$

b)



c)