

Name: _____
Student ID: _____

Test 3

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

- a. (5 marks) Sketch the graph of the piece-wise function:

$$f(x) = \begin{cases} (x+1)^2 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x < 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$$

- b. (2 marks) State the domain and range of the above function, $f(x)$.
c. (1 mark) Is the above function, $f(x)$, injective, justify.

Question 2.

- a. (4 marks) Sketch the graph of the following function:

$$f(x) = 2^{x+1} - 2$$

- b. (4 marks) Sketch the graph of the following function:

$$g(x) = \log_3(x+1) + 2$$

- c. (2 marks) State the domain and range of the above two functions, $f(x)$ and $g(x)$.
d. (2 marks) Are the above two functions, $f(x)$ and $g(x)$, injective, justify.

Question 3.

- a. (1 mark) Suppose $h(x)$ is injective and $h(2) = 3$, find $h^{-1}(3)$
- b. (4 marks) Verify that $g(x)$ is the inverse of $f(x)$, where $g(x) = \sqrt[3]{x+1}$ and $f(x) = x^3 - 1$.
- c. (5 marks) Find $k^{-1}(x)$ for the following function:

$$k(x) = \frac{2x+5}{3x-2}$$

Question 4.

- a. (2 marks) Express in terms of a simple logarithm:

$$\log \sqrt[4]{xy^2\sqrt{z}}$$

- b. (2 marks) Express as a single logarithm with coefficient of one:

$$\frac{1}{2} \log x + \frac{3}{2} \log 2y - \log x^2y$$

- c. (5 marks) Solve for x .

$$\ln(x-2) + \ln(2x+1) = 2 \ln x$$

Question 5. Let $f(x) = \log_2 x$ then

- a. (1 mark) evaluate $f(0)$.
- b. (1 mark) evaluate $f(1)$.
- c. (1 mark) evaluate $f\left(\frac{1}{8}\right)$.

Question 6. Solve for x .

- a. (1 mark)

$$\log_x 3 = -1$$

- b. (4 marks)

$$4^{2x-1} = 7^{x+2}$$

Bonus.

- a. (2 marks) Solve for x :

$$[\log_3(x-1)]^2 - 3\log_3(x-1) = 4$$

- b. (2 marks) Solve for x :

$$2^x + 2^{-x} = 2$$

- c. (2 marks) Solve for x :

$$\ln x - \log x = 1$$