NAME: SOLUTIONS **MARK: /50**

TEST 2 – Version 1
201-BZS-05
Probability & Statistics
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Date: March 6th 2009

Question 1 (7 marks 2 + 5)

(a) In how many different ways can we seat 10 class mates in a row of 15 chairs?

(b) In how many ways can we seat 5 girls and 5 boys in a row of 15 chairs, if at least 3 girls must always be seated right next to each other?

(a) $\binom{15}{10} {}_{10}P_{10} = \frac{15!}{10!\,5!}(10!) = \frac{15!}{5!}$

(b) $\binom{13}{8}\binom{5}{3}\, 3!\quad 8! = 3113510400$

Regroup 3 chairs together

Pick 3 of 5 girls

Ways of seating 3 girls in group

Ways of seating 7 people & (group of 3 people counting as 1)

ANOTHER WAY TO THINK ABOUT IT: $\frac{13!}{5!}\binom{5}{3}3!$

Your are arranging 8 distinct items (7 people + group of 3 girls) & 5 empty chair (that are identical)

There are $\frac{13!}{5!}$ ways to do that; multiply by $\binom{5}{3} \cdot 3!$

Question 2 (9 marks 1 + 2 + 3 + 3)

A multiple choice test is made up of 15 questions. Each question has 4 possible choices (1 right and 3 wrong). If a student randomly guesses the answers to all 15 questions, find the probability that the student:

(a) Gets all 15 questions right

(b) Gets all 15 questions wrong

(c) Gets at least 2 questions right

(d) Gets exactly 8 questions right

(a) $\dfrac{1}{3^{15}}$ (b) $\dfrac{2^{15}}{3^{15}}$

(c) At least 2 right

$= 1 - P(0 \text{ Right}) - P(1 \text{ right})$

$= 1 - \dfrac{2^{15}}{3^{15}} - \dfrac{\binom{15}{1} 2^{14}}{3^{15}}$

(d) $P(8) = \dfrac{\binom{15}{8} 2^{7}}{3^{15}}$

Question 3 (7 marks)
Four individuals with unknown blood types are to give blood. They will each have their blood tested one at a time. We know that only one of them has the desired blood type O+. What is the probability that we must test 3 individuals before finding the one with the desired blood type?

Let A_1 = PERSON 1 is NOT O+

A_2 = Person 2 is NOT O+

$$P(\text{TESTING } 3) = P(A_1 \cap A_2)$$
$$= P(A_2 | A_1) \cdot P(A_1)$$
$$= \frac{2}{3} \cdot \frac{3}{4}$$
$$= \frac{1}{2}$$

Because IF THE FIRST 2 FAIL then the ANSWER will be determined on the 3rd person)

Question 4 (10 marks 2 + 3 + 3 + 2)

Consider a standard deck of 52 cards:

A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥

A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠

A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦

A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣

What is the probability of being dealt the following 6-card hands?

(a) A hand containing exactly one suit

(b) A hand consisting of exactly 2 suits

(c) A hand containing 3 pairs

(d) A hand containing 3 cards of the same face value and 3 cards all of different face values

(a) CHOOSE SUIT CHOOSE 6 CARDS IN SUIT

$$\frac{\binom{4}{1}\binom{13}{6}}{\binom{52}{6}} = 0.000337$$

(b) CHOOSE 2 SUITS CHOOSE 6 CARDS FROM THOSE 2 SUITS All hands with 1 SUIT only (FROM THE 2 SUITS)

$$\frac{\binom{4}{2}\binom{26}{6} - \binom{13}{6}\cdot 2}{\binom{52}{6}}$$

$$= 0.06768$$

(c) ← CHOOSE FACES OF the 3 pairs ← CHOOSE which SUIT the cards will be

$$\frac{\binom{13}{3}\binom{4}{2}\binom{4}{2}\binom{4}{2}}{\binom{52}{6}}$$

$$= 0.00303$$

(d) ← CHOOSE THE TRIPLE ← CHOOSE THE THREE singles ← CHOOSE SUITS

$$\frac{\binom{13}{1}\binom{12}{3}\binom{4}{3}\binom{4}{1}^3}{\binom{52}{6}} = 0.03596$$

Question 5 (9 marks 3 + 3 + 3)

A certain population is asked about their taste in artichokes, broccoli and cauliflower. Consider the following events and probabilities:

A = likes artichokes

B = likes broccoli

C = likes cauliflower

Consider the following probabilities:

$P(A) = 0.31$ $P(B) = 0.26$ $P(C) = 0.15$

$P(A \cap B) = 0.1$ $P(B \cap C) = 0.04$ $P(A \cap C) = 0.05$

$P(A \cap B \cap C) = 0.00$

Find the probability of the following events.

Accompany each event with its corresponding Venn Diagram.

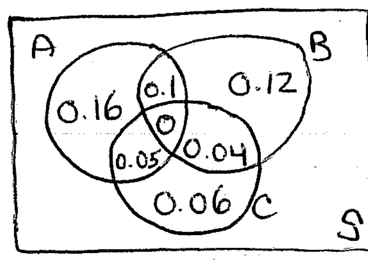
(a) The event that a person likes exactly one of the three types of vegetables

(b) The event that a person likes at least one vegetable

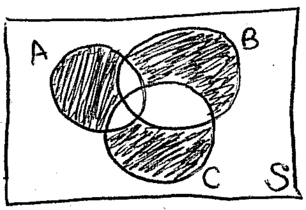
(c) The event that a person likes more than one vegetable, given that we know they like broccoli or cauliflower

*BONUS (2 marks) Express the event in (a) symbolically, that is, using only (A, B, C, ∩, U, ')

SUMMARY



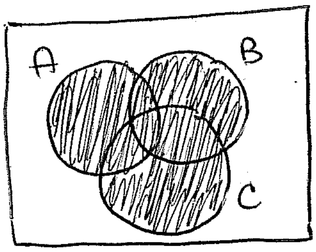
(a)



$P(\text{EXACTLY } 1) = 0.16 + 0.12 + 0.06$

$= 0.34$

(b)

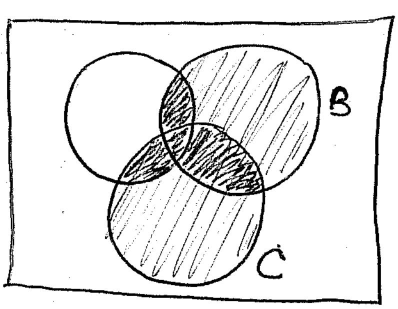


$P(\geq 1) = P(A \cup B \cup C)$

$= 0.16 + 0.1 + 0.05 + 0.04 + 0.12 + 0.06$

$= 0.53$

(c)



$P(\text{MORE than } 1 \mid B \cup C)$

$= \dfrac{0.19}{0.37}$

Question 6 (8 marks 4 + 4)

Seventy percent of the light aircraft that disappear while in flight over Iraq are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.

Denote the events as (D) Discovered (D') Not discovered (L) Locator (L') No locator

(a) What is the probability that an aircraft that disappears while in flight over Iraq has a locator, that is, what is P(L)?

(b) If it has an emergency locator, what is the probability that it will not be discovered?

WHAT DATA DO WE HAVE ?

$P(D) = 0.7 \qquad P(D') = 0.3$

$P(L|D) = 0.6 \qquad P(L'|D) = 0.4$

$P(L'|D') = 0.9 \qquad P(L|D') = 0.1$

(a) $P(L) = P((L \cap D) \cup (L \cap D'))$

$= P(L \cap D) + P(L \cap D')$

$= P(L|D) \cdot P(D) + P(L|D') \cdot P(D')$

$= (0.6)(0.7) + (0.1)(0.3)$

$= 0.45$

(b) $P(D'|L) = \dfrac{P(D' \cap L)}{P(L)} = \dfrac{(0.1)(0.3)}{0.45} \approx 0.0667$