NAME: SOLUTIONS **MARK: /50**

TEST 2 – Version 2
201-BZS-05
Probability & Statistics
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Date: March 13th 2009

Question 1 (8 marks 1 + 2 + 2 + 3)
A 5-member committee is to be picked from a group of 3 Green Party members, 7 Marxist-Leninists and 5 Communists.
In how many ways can the committee be formed if it is to contain
(a) all Communists?
(b) at least one Green Party member?
(c) all members of the same party?
(d) at least one Marxist-Leninist and one Communist?

(a) $\binom{5}{5} = 1$ way

(b) ALL possible COMMITTEES − COMMITTEES WITH NO green PARTY MEMBERS

$= \binom{15}{5} - \binom{12}{5} = 2211$ ways

(c) IT CAN BE EITHER ALL MARXISTS OR ALL COMMUNISTS

$\binom{7}{5} + \binom{5}{5} = 22$ ways

(d) ALL COMMITTEES − COMMITTEES WITH NO MARXIST & AT LEAST 1 COMMU. − COMMITTEES WITH NO COMM. & AT LEAST 1 MARXIST − $\underbrace{\text{COMMIT. WITH NO MARXIST NOR COMMUNIST}}_{\text{impossible}}$

$= \binom{15}{5} - \binom{10}{5} - \binom{8}{5}$

$=$

$= 2695$ ways

Question 2 (8 marks 1 + 2 + 2 + 3)
A couple intends to have 10 children. Assuming that having a boy or having a girl are equally likely events, what is the probability that they have
(a) 10 boys?
(b) 5 boys and 5 girls?
(c) At least 3 girls?
(d) At least 7 children of one sex?

$x = \#$ OF girls $\quad n = 10$
$p = \frac{1}{2} \quad q = \frac{1}{2}$

(a) $\binom{10}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}} = 0.000977$

(b) $\binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = 0.246$

(c) $1 - P(x=2) - P(x=1) - P(x=0)$

$= 1 - \binom{10}{2}\left(\frac{1}{2}\right)^{10} - \binom{10}{1}\left(\frac{1}{2}\right)^{10} - \binom{10}{0}\left(\frac{1}{2}\right)^{10}$

$= 1 - 0.05468$

$= 0.9453$

(d) $\left[P(x=7) + P(x=8) + P(x=9) + P(x=10)\right] \times 2$
$\uparrow$ FOR BOTH sexes

$= \left[\binom{10}{7}\left(\frac{1}{2}\right)^{10} + \binom{10}{8}\left(\frac{1}{2}\right)^{10} + \binom{10}{9}\left(\frac{1}{2}\right)^{10} + \binom{10}{10}\left(\frac{1}{2}\right)^{10}\right] \cdot 2$

$= 0.34399$

Question 3 (8 marks)
Marie is getting married tomorrow, at an outdoor ceremony in the desert. Here, it rains on average only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Let R = ACTUAL RAINS
R' = ACTUALLY does NOT RAIN

F = WEATHERMAN FORECASTS RAIN

F' = " " does NOT FORCAST RAIN

We KNOW the FOLLOWING

$P(R) = 5/365 \qquad P(R') = 360/365$

$P(F|R) = 0.9 \qquad P(F|R') = 0.10$

We WANT $P(R|F)$

$$P(R|F) = \frac{P(R\cap F)}{P(F)} = \frac{P(R\cap F)}{P(F\cap R) + P(F\cap R')}$$

$$= \frac{P(F|R)\cdot P(R)}{P(F|R)\cdot P(R) + P(F|R')\cdot P(R')}$$

$$= \frac{(0.9)(5/365)}{(0.9)(5/365) + (0.1)\left(\frac{360}{365}\right)}$$

$$= \frac{0.012}{0.1109} = 0.1111$$

Question 4 (7 marks 2.5 + 2 + 2.5)

Consider a standard deck of 52 cards:

A♥ 2♥ 3♥ 4♥ 5♥ 6♥ 7♥ 8♥ 9♥ 10♥ J♥ Q♥ K♥

A♠ 2♠ 3♠ 4♠ 5♠ 6♠ 7♠ 8♠ 9♠ 10♠ J♠ Q♠ K♠

A♦ 2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦

A♣ 2♣ 3♣ 4♣ 5♣ 6♣ 7♣ 8♣ 9♣ 10♣ J♣ Q♣ K♣

(A, 2, 3 . . . K are the face values) (♥ ♠ ♦♣ are the suits)

What is the probability of being dealt the following **7-card** hands?

(a) Four cards of one face value and 3 other cards all of the same suit?

(b) 2 pairs (pair = 2 cards of same face value) and 3 cards all of different face values?

(c) A hand consisting of exactly 2 suits?

(a) $$\frac{\binom{13}{1}\binom{4}{4}\binom{12}{3}\binom{4}{1}}{\binom{52}{7}} = 0.0000855$$

(b) $$\frac{\binom{13}{2}\binom{4}{2}^2\binom{11}{3}\binom{4}{1}^3}{\binom{52}{7}} = 0.222$$

(c) $$\frac{-\binom{4}{1}\binom{13}{7} + \binom{4}{2}\binom{26}{7}}{\binom{52}{7}} = 0.0294$$

Question 5 (4 marks)

In how many ways can we sit 6 girls and 5 boys in a row of 14 chairs if at least 4 boys must be sitting next to each other (with no empty chairs between them)?

GROUP 4 BOYS & CONSIDER THEM 1 PERSON
GROUP 4 CHAIRS TOGETHER & CONSIDER THEM 1 CHAIR.
SO THERE ARE NOW 11 CHAIRS & 8 "PEOPLE"

$$\underbrace{\binom{11}{8}}_{\text{WHICH SEATS ARE EMPTY}} \cdot \underbrace{\binom{5}{4}}_{\text{WHICH BOYS SIT TOGETHER}} \cdot \underbrace{8!}_{\text{ordering THE "people"}} \cdot \underbrace{4!}_{\text{ordering the 4 boys}} = 79833600 \text{ WAYS}$$

Question 6 (8 marks 2 + 2 + 2 + 2)

A certain population is asked whether they have travelled to (A) Argentina, (B) Bolivia or (C) Cambodia.

Consider the following probabilities:

$P(A) = 0.3$ $\quad P(B) = 0.26 \quad$ $P(C) = 0.15$

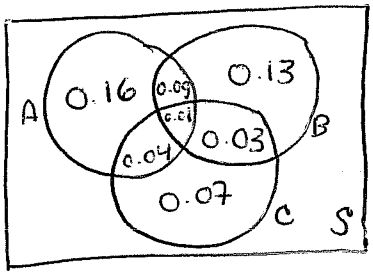
$P(A \cap B) = 0.1$ $\quad P(B \cap C) = 0.04 \quad$ $P(A \cap C) = 0.05$

$P(A \cap B \cap C) = 0.01$

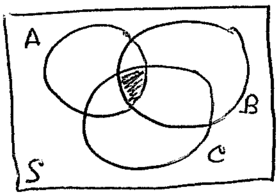
(a) What is the probability that a person has visited all three countries?
(b) What is the probability that a person has not visited any of the countries?
(c) What is the probability that a person has visited exactly 2 of the countries?
(d) If we know a person has been to at least one of the three countries, what is the probability that they have only been to Bolivia?

Note: Accompany questions (a)-(c) with Venn Diagrams in order to get full marks.

(a)

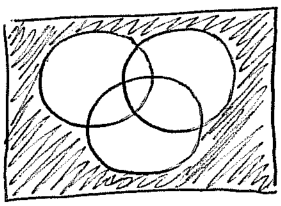


(a)



0.01

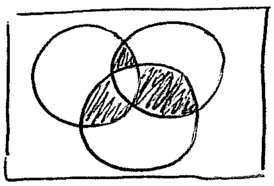
(b)



$1 - [0.16 + 0.09 + 0.13 + 0.04 + 0.01 + 0.03 + 0.07]$

$= 0.47$

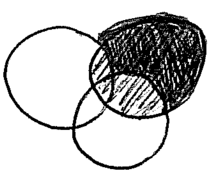
(c)



$0.04 + 0.09 + 0.03$

$= 0.16$

(d)



$P(\text{only } B \mid A \cup B \cup C)$

$= \dfrac{P(\text{only } B)}{P(A \cup B \cup C)} = \dfrac{0.13}{0.53} = 0.245$

Question 6 (7 marks)

A hospital receives 1/5 of its flu vaccines from Company X and the remainder from Company Y. For company X, 10% of the flu vaccine vials are ineffective per shipment, for Company Y, 2% of the vials are ineffective per shipment.

The hospital tests 30 randomly selected vials from one shipment and finds that one vial is ineffective. What are the chances that the shipment came from Company X?

$P(X) = 1/5 \qquad P(Y) = 4/5$

10% OF X defective $\qquad$ 2% OF Y DEFECTIVE

$P(X \mid 1/30 \text{ FROM X OR Y defective})$

$$= \frac{P(X \cap 1/30 \text{ FROM X or Y DefEC.})}{P(1/30 \text{ defective})} = \frac{P(1/30 \text{ def} \mid X) \cdot P(X)}{P(1/30 \text{ def} \mid X) \cdot P(X) + P(1/30 \text{ def} \mid Y) \cdot P(Y)}$$

$$= \frac{\left[\binom{30}{1}(0.1)^1(0.9)^{29}\right] 1/5}{\left[\binom{30}{1}(0.1)^1(0.9)^{29}\right] 1/5 + \left[\binom{30}{1}(0.02)^1(0.98)^{29}\right] 4/5}$$

$$= \frac{0.1413}{0.1413 + 0.267} = \boxed{0.346}$$

BONUS (2 marks)

Urn A contains 4 red and 6 blue. Urn B contains 16 red balls and x blue balls. A single ball is drawn from each urn. The probability that both balls are the same colour is 0.44. Calculate the number of blue balls x in urn B.

A: 4R, 6B $\qquad$ B: 16 R, xB

$$P(2\text{ Red}) + P(2\text{ Blue}) = 0.44$$

$$\left(\frac{4}{10}\right)\left(\frac{16}{16+x}\right) + \left(\frac{6}{10}\right)\left(\frac{x}{16+x}\right) = 0.44$$

$$64 + 6x = (10)(16+x)0.44$$

$$64 + 6x = 70.4 + 4.4x$$

$$x = 4$$

THERE ARE 4 blue BALLS