NAME: SOLUTIONS **MARK: /40**

TEST 3
201-BZS-05
Probability & Statistics
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Date: April 3rd 2009

Question 1 (5 marks)
The number of customers arriving per hour a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 4$.

(a) Compute the probability that more than 2 customers will arrive in a 1 hour period.
(b) Compute the probability that fewer customers than expected will arrive in a 30 minute period.

(a)
$$P(x>2) = 1 - P(x \leq 2)$$
$$= 1 - \left(P(x=2) + P(x=1) + P(x=0)\right)$$
$$= 1 - \left(\frac{e^{-4} \cdot 4^2}{2!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^0}{0!}\right)$$
$$= 0.762$$

(b) How many customers are expected in 30 minutes; $\lambda = 2$

$$P(x<2) = \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^0}{0!}$$
$$= 0.406$$

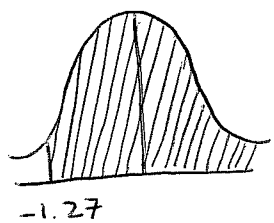
Question 2 (6 marks)

A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with a standard deviation of 6.3 months, find the probability that a given mouse will live

(a) More than 32 months
(b) Less than 28 months
(c) Between 37 and 49 months

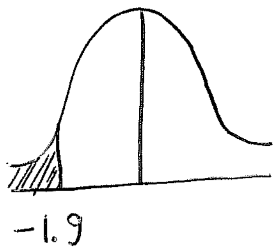
$\mu = 40 \quad \sigma = 6.3$

(a) $z = \dfrac{32-40}{6.3} = -1.27$



$P(x > 32)$
$= P(z > -1.27)$
$= 0.5 + 0.3980 = \boxed{0.898}$

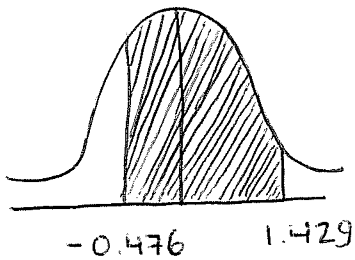
(b) $z = \dfrac{28-40}{6.3} = -1.9$



$P(x < 28)$
$= P(z < -1.9)$
$= 0.5 - 0.4713 = \boxed{0.0287}$

(c) $z = \dfrac{37-40}{6.3} = -0.476$

$z = \dfrac{49-40}{6.3} = 1.429$



$= 0.1822 + 0.4236$
$= \boxed{0.6058}$

Question 3 (6 marks)

The probability that a patient recovers from a rare blood disease is 0.06. If 100 people are known to have contracted this disease then

(a) State the EXACT probability that more than 3 people survive.

(b) State the APPROXIMATE probability that more than 3 people survive, justify why your approximation is valid.

(a) we have binomial with $n = 100$, $p = 0.06$

X = # of people who survive

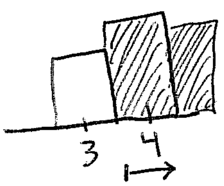
$$P(X > 3) = 1 - P(X \le 3)$$

$$= 1 - \left(\binom{100}{3}(0.06)^3(0.94)^{97} + \binom{100}{2}(0.06)^2(0.94)^{98} + \binom{100}{1}(0.06)^1(0.94)^{99} + \binom{100}{0}(0.94)^{100} \right)$$

$$= 1 - (0.0864 + 0.04144 + 0.0131 + 0.002)$$

$$= 1 - 0.1430 = \boxed{0.85698}$$

(b) we can use normal approximation since $np = 0.06(100) = 6 \geq 5$ & $n(1-p) = 0.94(100) = 94 \geq 5$



$P(X > 3) \Rightarrow$ continuity correction

$$\approx P(X \geq 3.5)$$

$$= P(Z \geq -1.05)$$

$$= 0.3508 + 0.5$$

$$= \boxed{0.8508}$$

$\mu = np = 100(0.06) = 6$

$\sigma = \sqrt{np(1-p)} = 2.375$

$$Z = \frac{3.5 - 6}{2.375} = -1.05$$

Question 4 (6 marks)

Consider an urn containing 100 marbles, 70 blue ones and 30 red ones. Five marbles are selected from the urn without replacement.

(a) What is the probability of selecting exactly 4 blue marbles?

(b) Use the binomial distribution to compute the approximate probability of selecting 4 blue marbles. What condition do we require for this approximation to be valid?

(c) What assumption is made when using the binomial in (b)?

(a) $$P = \frac{\binom{70}{4}\binom{30}{1}}{\binom{100}{5}} = 0.365$$

(b) we require that n/N is $\leq$ than 0.05

$\left(5/100 = 0.05\right)$ so we are fine

$P = 0.7$

$$P(4\text{ blue}) = \binom{5}{4}(0.7)^4(0.3)^1 = 0.36015$$

(c) THAT we are SAMPLING with replacement, IN THE BINOMIAL.

whereas in reality there is NO replacement

because PART (a) is the Hypergeometric CASE

Question 5 (6 marks)

(a) Consider a normal distribution of random variable x and mean $\mu = 45$.
Find σ if $P(x < 47.5) = 0.9495$.

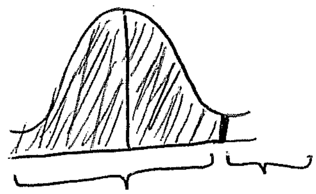
(b) Consider a normal distribution of random variable x with $\sigma = 9$.
Find μ if $P(x > 65) = 0.6915$.

(c) Consider the standard normal distribution. Find z_0 if $P(-z_0 > z) = 0.2266$.

(a) Z value corresponding to an area of 0.9495 is 1.64 ; $Z = \frac{x-\mu}{\sigma}$

$$1.64 = \frac{47.5-45}{\sigma}$$

$$\boxed{\sigma = 1.524}$$

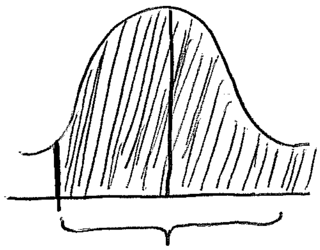


(b) Z value corresponding to area 0.6915 is

$Z = -0.5$

$$Z = \frac{x-\mu}{\sigma}$$

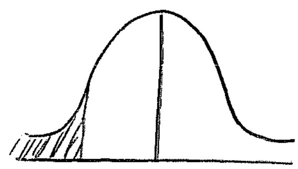
$$-0.5 = \frac{65-\mu}{9} \qquad \boxed{\mu = 69.5}$$



(c)

$$P(-Z_0 > Z) = 0.2266$$

$$\boxed{Z_0 = 0.75}$$



Question 6 (6 marks)

The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assume that x, the lives of these machines follow approximately a normal distribution.

(a) Find the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years.

(b) A sample of nine machines yields the following values for x:

5.6 7.2 6.5 2.3 6.8 6 7.1 5.9 6.9

Find the sample mean.

(c) What is the probability of getting a sample mean smaller than the one computed in (b)?

(a) We are dealing with $P_{\bar{x}}$ which is Approx. Normal since the parent population P_x is.

$\mu = 7 \quad \sigma = 1$

$$Z_1 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{6.4-7}{1/\sqrt{9}} = -1.8$$

$$Z_2 = \frac{7.2-7}{1/\sqrt{9}} = 0.6$$

so $P(6.4 \le \bar{x} \le 7.2)$

$= P(-1.8 \le Z \le 0.6)$

$= 0.4641 + 0.2257 = \boxed{0.6898}$

(b) $\bar{x} = \Sigma x/n = \boxed{6.033}$

(c) $P(\bar{x} < 6.033)$

$$Z = \frac{6.033-7}{1/\sqrt{9}} = -2.9$$

$P(Z < -2.9) = 0.5 - 0.4981$

$= \boxed{0.0019}$

Question 7 (5 marks)

Consider a 5 question multiple choice test with 3 wrong answers and 1 correct answer for each question. Consider the random variable, x, defined by the number of correct answers on the test.

(a) Give the probability distribution for x

(b) Find μ and σ^2

(a)

x	$P(x)$
0	$\binom{5}{0}(0.25)^0(0.75)^5 = 0.237$
1	$\binom{5}{1}(0.25)^1(0.75)^4 = 0.3955$
2	$\binom{5}{2}(0.25)^2(0.75)^3 = 0.2637$
3	$\binom{5}{3}(0.25)^3(0.75)^2 = 0.0879$
4	$\binom{5}{4}(0.25)^4(0.75)^1 = 0.01465$
5	$\binom{5}{5}(0.25)^5(0.75)^0 = 0.000977$

(b) $\mu = \sum x \cdot P(x) = 1.25 \quad (np = 5 \cdot (0.25) = 1.25)$

$\sigma^2 = \sum x^2 \cdot P(x) - \mu^2 = 2.5 - (1.25)^2 = 0.9375$

BONUS (2 marks)

Give a brief explanation of the Central Limit Theorem

For any population, no matter what its distribution, with mean μ & s.d σ. As n becomes larger (the sample size) the sampling distribution of the population of sample means $\bar{x}$, becomes more normal with mean μ & s.d $\sigma/\sqrt{n}$.