Dawson College: Calculus II: 201-NYB-05 12

Due April 26, 2010

Last Name: ____________________

First Name: ____________________

Student ID: ____________________

Quiz/Assignment 10

Question 1. Use the disc/washer method to find the volume of the solid obtained by rotating:
(a) *(5 marks)* the region bounded by $y = x^2/4$ and $y = 5 - x^2$ about the x-axis.
(b) *(5 marks)* the region bounded by $y = 1/x$, $y = 0$, $x = 1$, $x = 3$ about $y = -1$.
(c) *(5 marks)* the region bounded by about $y = x$ and $y = \sqrt{x}$ about $x = 2$.

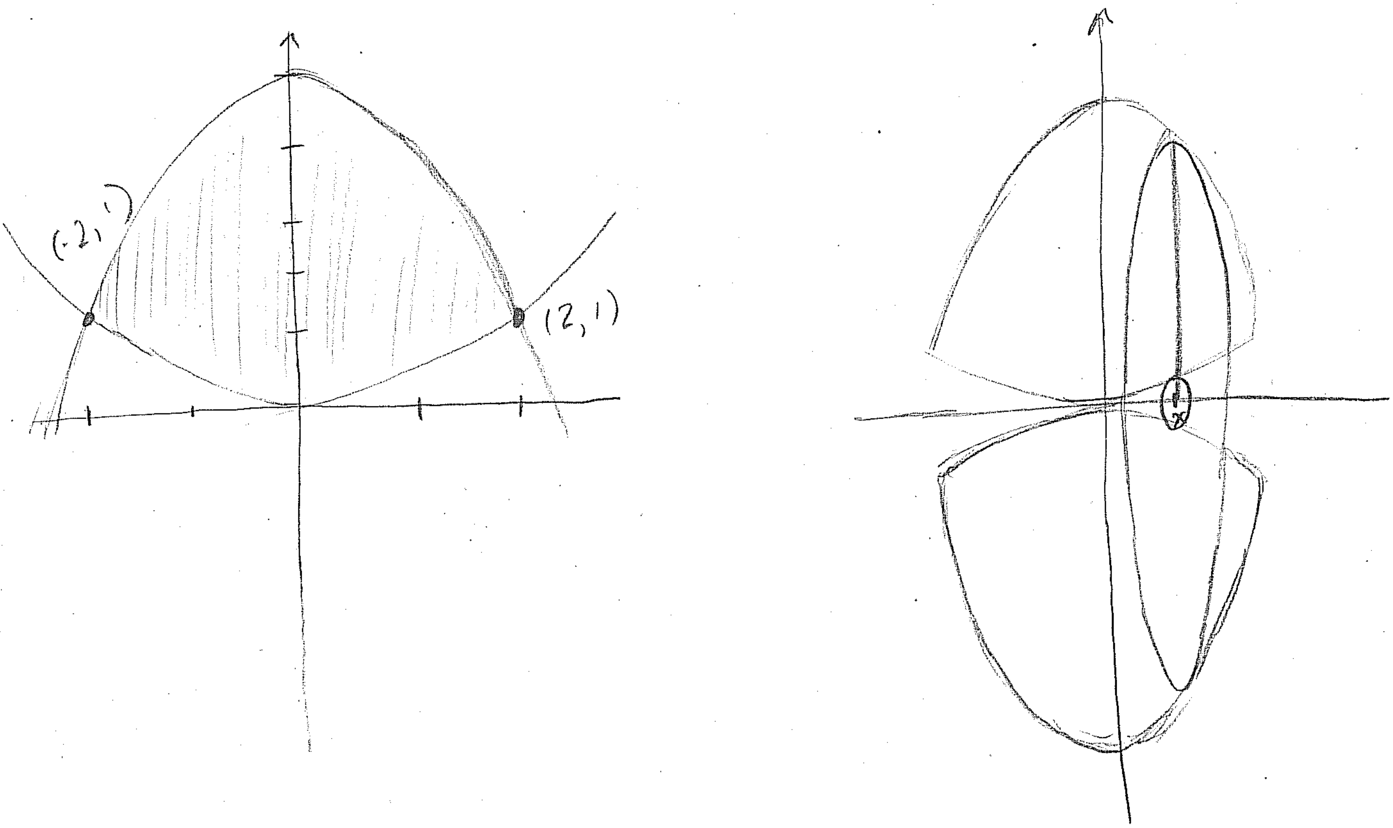
Question 2. Use the method of cylindrical shells to find the volume of the solid obtained by rotating:
(a) *(5 marks)* the region bounded by $y = 3 + 2x - x^2$ and $x + y = 3$ about the y-axis.
(b) *(5 marks)* the region bounded by $x + y = 3$ and $x = 4 - (y-1)^2$ about the x-axis.
(c) *(5 marks)* the region bounded by $y = 4x - x^2$, $y = 8x - 2x^2$ about $x = -2$.

QUIZ / ASSIGNMENT 10

SOLUTIONS

1a) INTERSECTION: $\frac{1}{4}x^2 = 5 - x^2 \Rightarrow x^2 = 20 - 4x^2$

$\Rightarrow 5x^2 = 20 \Rightarrow x^2 = 4 \Rightarrow x = \pm 4$



$A(x) = \pi R^2 - \pi r^2$, $R = 5 - x^2$, $r = \frac{1}{4}x^2$

$\therefore A(x) = \pi(5-x^2)^2 - \pi\left(\frac{1}{4}x^2\right)^2$

$\therefore V = \int_{-2}^{2} \pi(5-x^2)^2 - \pi\left(\frac{1}{4}x^2\right)^2 \, dx$

$= \pi \int_{-2}^{2} 25 - 10x^2 + x^4 - \frac{1}{16}x^4 \, dx$

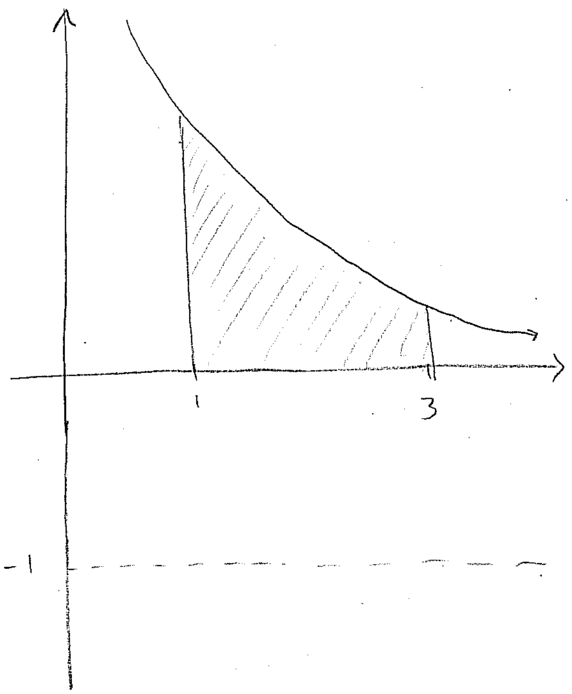
$= \pi \int_{-2}^{2} 25 - 10x^2 + \frac{15}{16}x^4 \, dx$

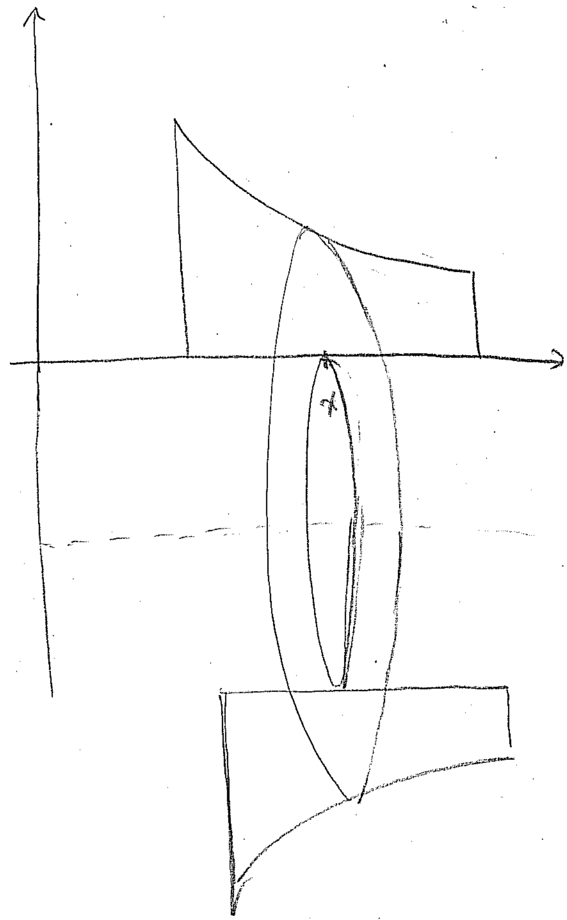
$$= \pi \left[25x - \frac{10}{3}x^3 + \frac{3}{16}x^5 \right]_{-2}^{2}$$

$$= \pi \left[\left(25(2) - \frac{10}{3}(2)^3 + \frac{3}{16}(2)^5\right) - \left(25(-2) - \frac{10}{3}(-2)^3 + \frac{3}{10}(-2)^5\right) \right]$$

$$= \frac{176}{3}\pi \text{ units}^3$$

5)





$A(x) = \pi R^2 - \pi r^2$

$R = 1 + \frac{1}{x} \qquad r = 1$

$\therefore A(x) = \pi\left(1+\frac{1}{x}\right)^2 - \pi(1)^2$

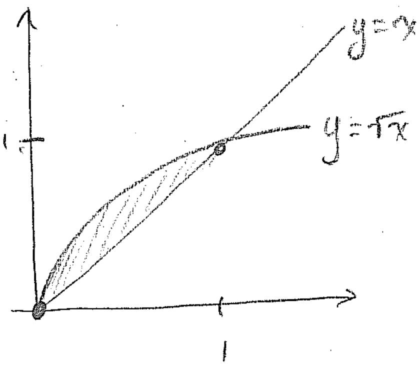
$$\therefore V = \int_1^3 \pi\left(1+\frac{1}{x}\right)^2 - \pi \, dx$$

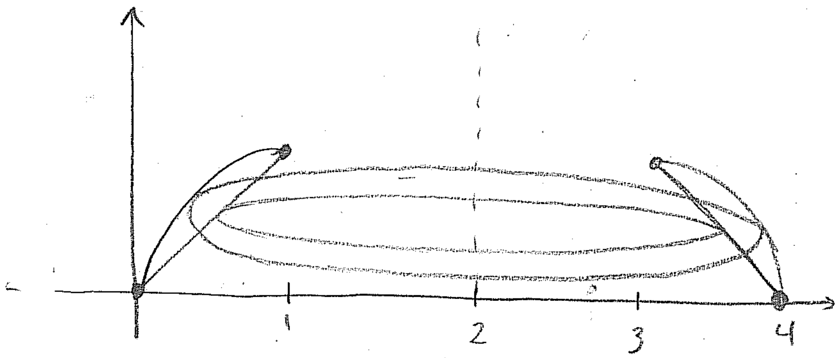
$$= \pi \int_1^3 \left(1 + \frac{2}{x} + \frac{1}{x^2} - 1\right) dx = \pi \int_1^3 \frac{2}{x} + \frac{1}{x^2} \, dx$$

$= \pi \left[2\ln|x| - \frac{1}{x} \right]_1^3 = \pi \left[\left(-\frac{1}{3} + 2\ln 3\right) - (-1 + 0) \right]$

$= \pi \left(2\ln 3 + \frac{2}{3} \right)$ units^3

c)





$A(y) = \pi R^2 - \pi r^2$, $R = 2 - y^2$ $r = 2 - y$

$\therefore A(y) = \pi (2 - y^2)^2 - \pi (2 - y)^2$

$\therefore V = \int_0^1 \pi (2 - y^2)^2 - \pi (2 - y)^2 \, dy$

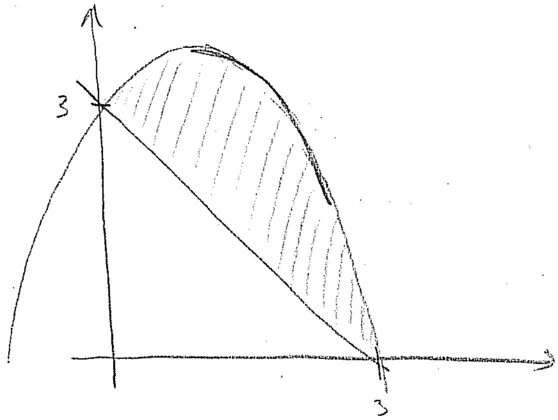
$= \pi \int_0^1 (4 - 4y^2 + y^4) - (4 - 2y + y^2) \, dy$

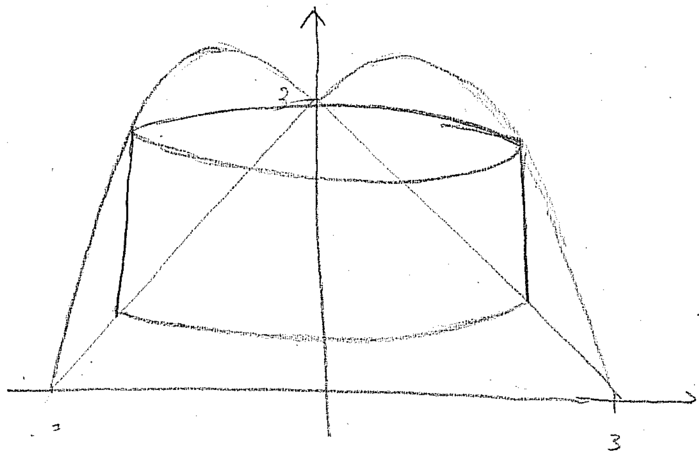
$= \pi \int_0^1 y^4 - 5y^2 + 4y \, dy = \pi \left[\frac{1}{5} y^5 - \frac{5}{3} y^3 + 2y^2 \right]_0^1$

$= \pi \left(\frac{1}{5} - \frac{5}{3} + 2 \right) = \frac{8}{15} \pi$

2) a) INTERSECTION: $3+2x-x^2 = 3-x \Rightarrow 3x-x^2=0$

$\Rightarrow x(3-x)=0 \Rightarrow x=0,3$





$A(x) = 2\pi r h, \quad r = x, \quad h = (3+2x-x^2)-(3-x)$

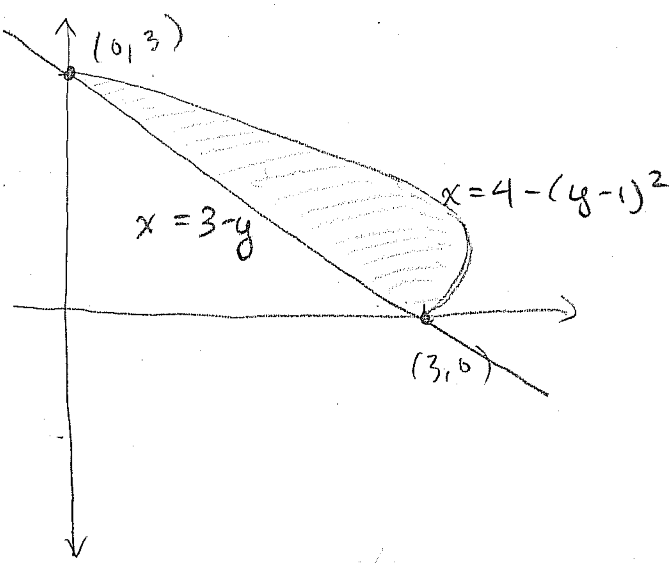
$= 3x - x^2$

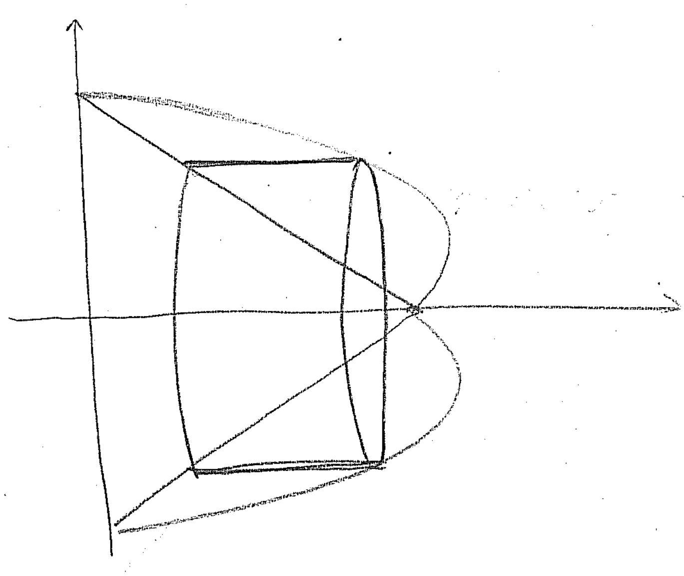
$\therefore \; V = \int_0^3 2\pi x(3x-x^2)\,dx = 2\pi \int_0^3 3x^2 - x^3\,dx$

$= 2\pi\left[x^3 - \frac{1}{4}x^4\right]_0^3 = 2\pi\left(27 - \frac{81}{4}\right)$

$= \frac{27}{2}\pi$ units^3

b)





$A(y) = 2\pi r h \qquad r = y, \qquad h = 4-(y-1)^2 - (3-y)$

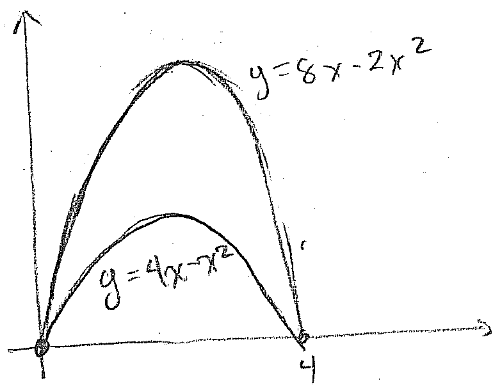
$$= 4-(y^2-2y+1) - 3 + y$$
$$= 4 - y^2 + 2y - 1 - 3 + y$$
$$= -y^2 + 3y$$

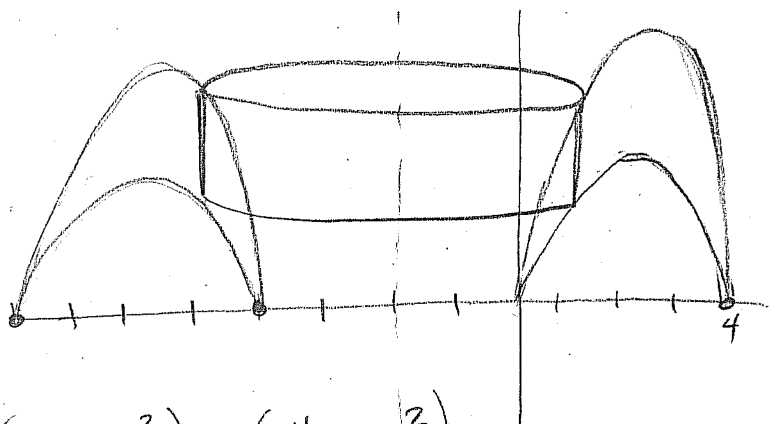
$$\therefore V = \int_0^3 2\pi y(-y^2+3y)\,dy = 2\pi \int_0^3 -y^3 + 3y^2\,dy$$

$$= 2\pi \left[-\tfrac{1}{4}y^4 + y^3\right]_0^3 = 2\pi\left(-\tfrac{81}{4} + 27\right) = \tfrac{27}{2}\pi \text{ units}^3$$

c) INTERSECTION: $8x - 2x^2 = 4x - x^2 \Rightarrow 4x - x^2 = 0 \Rightarrow x(4-x) = 0$

$\Rightarrow x = 0, 4$





$A(x) = 2\pi r h, \qquad r = 2+x \quad h = (8x-2x^2) - (4x-x^2)$
$$= 4x - x^2$$

$$\therefore V = \int_0^4 2\pi(2+x)(4x-x^2)\,dx$$

$$= 2\pi \int_0^4 8x + 2x^2 - x^3\,dx = 2\pi\left[4x^2 + \tfrac{2}{3}x^3 - \tfrac{1}{4}x^4\right]_0^4$$

$$= 2\pi\left(64 + \frac{128}{3} - 64\right)$$

$$= \frac{256}{3}\pi \text{ cm}^3$$