

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 4

Question 1. Evaluate the following:

(a) (4 marks)

$$\frac{d}{dx} \int_{e^x}^0 \sin^3 t dt = \frac{d}{dx} \left[- \int_0^{e^x} \sin^3 t dt \right]$$

$$= - \frac{d}{dx} \left[\int_0^{e^x} \sin^3 t dt \right] = - \frac{d}{dx} [f(g(x))]$$

$$= - f'(g(x)) \cdot g'(x) = - \sin^3 e^x \cdot e^x$$

$$= - e^x \sin^3 e^x$$

where

$$f(x) = \int_0^x \sin^3 t dt$$

$$g(x) = e^x$$

$$f'(x) = \sin^3 x$$

$$g'(x) = e^x$$

(b) (2 marks)

$$\int \sqrt{\cot x} \csc^2 x dx = \int \sqrt{u} \csc^2 x \frac{du}{-\csc^2 x}$$

$$= - \int u^{1/2} du = - \frac{2}{3} u^{3/2} + C$$

$$= - \frac{2}{3} \cot^{3/2} x + C$$

Let $u = \cot x$

$$du = -\csc^2 x dx$$

$$dx = \frac{du}{-\csc^2 x}$$

(c) (4 marks)

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int_1^2 \frac{e^u}{\frac{1}{\sqrt{x}}} 2\sqrt{x} dx$$

$$= 2 \int_1^2 e^u du = 2 [e^u]_1^2$$

$$= 2e^2 - 2e$$

Let $u = \sqrt{x}$

$$du = \frac{1}{2} x^{-1/2} dx$$

$$dx = 2\sqrt{x} du$$

If $x = 1 \Rightarrow u = 1$

$x = 4 \Rightarrow u = 2$