

Quiz 6

Question 1. Evaluate the following integrals:

(a) (5 marks)

$$\int_0^{\pi/4} \tan^3 x \sec^4 x dx$$

$$= \int_0^{\pi/4} (\underbrace{\sec^2 x}_{u^2} - 1) \underbrace{\sec^3 x}_{u^3} \underbrace{\sec x \tan x}_{du} dx$$

$$= \int_1^{\sqrt{2}} (u^2 - 1)u^3 du = \int_1^{\sqrt{2}} (u^5 - u^3) du$$

$$= \left[\frac{u^6}{6} - \frac{u^4}{4} \right]_1^{\sqrt{2}} = \left[\frac{(\sqrt{2})^6}{6} - \frac{(\sqrt{2})^4}{4} \right] - \left[\frac{(1)^6}{6} - \frac{(1)^4}{4} \right] = \frac{8}{6} - 1 - \frac{1}{6} + \frac{1}{4}$$

$$= \frac{5}{12}$$

(b) (5 marks)

$$\int x^3 \sqrt{9-x^2} dx$$

$$= \int (3 \sin \theta)^3 \cdot 3 \cos \theta \cdot 3 \cos \theta d\theta$$

$$= 243 \int \sin^3 \theta \cos^2 \theta d\theta$$

$$= 243 \int (1 - \cos^2 \theta) \cos^2 \theta \sin \theta d\theta$$

$$= -243 \int (1 - u^2) u^2 du = -243 \int (u^2 - u^4) du$$

$$= -243 \left(\frac{u^3}{3} - \frac{u^5}{5} \right) + C = -\frac{243}{3} \cos^3 \theta + \frac{243}{5} \cos^5 \theta + C$$

$$= -81 \left(\frac{\sqrt{9-x^2}}{3} \right)^3 + \frac{243}{5} \left(\frac{\sqrt{9-x^2}}{3} \right)^5 + C$$

LET $u = \sec x$

$du = \sec x \tan x dx$

IF $x=0 \Rightarrow u = \sec 0 = 1$

$x = \frac{\pi}{4} \Rightarrow u = \sec \frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

LET $x = 3 \sin \theta$ ON $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$dx = 3 \cos \theta d\theta$

AND

$\sqrt{9-x^2} = \sqrt{9-9\cos^2 \theta} = 3\sqrt{1-\cos^2 \theta}$

$= 3\sqrt{\sin^2 \theta} = 3|\sin \theta| = 3 \sin \theta$

SINCE $\cos \theta \geq 0$

LET $u = \cos \theta$

$du = -\sin \theta d\theta$

