P 299

#28)

$$\int \sin t \sec^2(\cos t)\,dt$$

LET $u = \cos t$

$du = -\sin t\, dt$

$dt = \dfrac{du}{-\sin t}$

$$= \int \cancel{\sin t} \sec^2 u \frac{du}{-\cancel{\sin t}}$$

$$= -\int \sec^2 u\, du = -\tan u + c$$

$$= -\tan(\cos t) + c$$

34)

$$\int \frac{x}{1+x^4}\,dx$$

LET $u = x^2$

$du = 2x\,dx$

$\therefore dx = \dfrac{du}{2x}$

$$= \int \frac{\cancel{x}}{1+u^2}\frac{du}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{2}\arctan u + C$$

$$= \tfrac{1}{2}\arctan x^2 + c$$

#46

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$$

LET $u = 1+2x$

$du = 2\,dx \Rightarrow dx = \frac{du}{2}$

AND $u = 1+2x$

$\Rightarrow u-1 = 2x$

$\Rightarrow x = \frac{u-1}{2}$

IF $x=0 \Rightarrow u=1$

$x=4 \Rightarrow u=9$

$$= \int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\,\frac{du}{2}$$

$$= \frac{1}{4}\int_1^9 \frac{u-1}{u^{1/2}}\,du$$

$$= \frac{1}{4}\int_1^9 \left(u^{1/2} - u^{-1/2}\right) du = \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$= \frac{1}{4}\left[\left(\frac{2}{3}(9)^{3/2} - 2(9)^{1/2}\right) - \left(\frac{2}{3}(1)^{3/2} - 2(1)^{1/2}\right)\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}(27) - 2(3) - \frac{2}{3} + 2\right]$$

$$= \frac{1}{4}\left[18 - 6 - \frac{2}{3} + 2\right] = \frac{1}{4}\left[14 - \frac{2}{3}\right]$$

$$= \frac{1}{4}\left[\frac{40}{3}\right] = \frac{10}{3}$$

#52)

$$\text{AVERAGE VALUE} = \frac{1}{2-0}\int_0^2 x^2\sqrt{1+x^3}\,dx$$

LET $u = 1+x^3$

$du = 3x^2 dx$

$dx = \dfrac{du}{3x^2}$

IF $x=0 \Rightarrow u=1$

$x=2 \Rightarrow u=9$

$$= \frac{1}{2}\int_1^9 \cancel{x^2}\sqrt{u}\,\frac{du}{3\cancel{x^2}}$$

$$= \frac{1}{6}\int_1^9 u^{1/2}\,du = \frac{1}{6}\left[\frac{2}{3}u^{3/2}\right]_1^9$$

$$= \frac{1}{6}\cdot\frac{2}{3}\left[9^{3/2} - 1^{3/2}\right] = \frac{1}{9}\left[27-1\right] = \frac{26}{9}$$

P309

#6) $\displaystyle\int t\sin 2t\,dt$

LET $u = t$ $\quad dv = \sin 2t\,dt$

$du = dt$ $\quad v = -\frac{1}{2}\cos 2t$

(INTEGRATE USING A SUBSTITUTION)

$$= uv - \int v\,du$$

$$= -\frac{t}{2}\cos 2t - \int\left(-\frac{1}{2}\cos 2t\right)dt$$

$$= -\frac{t}{2}\cos 2t + \frac{1}{2}\int\cos 2t\,dt$$

$$= -\frac{t}{2}\cos 2t + \frac{1}{2}\left[-\frac{1}{2}\sin 2t\right] + C$$

$$= -\frac{t}{2}\cdot\cos 2t - \frac{1}{4}\sin 2t + C$$

#16)

$$\int_0^1 (x^2+1)e^{-x}\,dx$$

LET

$u = x^2+1 \qquad dv = e^{-x}dx$

$du = 2x\,dx \qquad v = -e^{-x}$

$$= [uv]_0^1 - \int_0^1 v\,du$$

$$= \left[(x^2+1)(-e^{-x})\right]_0^1 - \int_0^1 (-e^{-x})\,2x\,dx$$

$$= \left[(2)(-e^{-1}) - (1)(-e^{0})\right] + 2\int_0^1 x e^{-x}\,dx$$

LET $u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = -e^{-x}$

$$= \left[-2e^{-1} + 1\right] + 2\int_0^1 x e^{-x}\,dx$$

$$= -2e^{-1} + 1 + 2\left[[uv]_0^1 - \int_0^1 v\,du\right]$$

$$= -2e^{-1} + 1 + 2\left(\left[x(-e^{-x})\right]_0^1 - \int_0^1 (-e^{-x})\,dx\right)$$

$$= -2e^{-1} + 1 + 2\left(\left[-e^{-1} - 0\right] + \int_0^1 e^{-x}\,dx\right)$$

$$= -2e^{-1} + 1 + 2\left(-e^{-1} + \left[-e^{-x}\right]_0^1\right)$$

$$= -2e^{-1} + 1 + 2\left(-e^{-1} + \left[-e^{-1} - (-e^{0})\right]\right)$$

$$= -2e^{-1} + 1 + 2\left(-e^{-1} - e^{-1} + 1\right) = -6e^{-1} + 3$$