April 22, 2010

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Quiz 10 (A)

Question 1. *(4 marks)* Do the planes $2x - 3y + 8z - 2 = 0$ and $3x + 5y + 9z - 12 = 0$ intersect? If so find the intersection.

$\vec{n_1} = (2, -3, 8)$ AND $\vec{n_2} = (3, 5, 9)$ ∴ THE PLANES ARE NOT PARALLEL

∴ THE PLANES INTERSECT.

$$\begin{bmatrix} 2 & -3 & 8 & 2 \\ 3 & 5 & 9 & 12 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 2 & -3 & 8 & 2 \\ 1 & 8 & 1 & 10 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 0 & -19 & 6 & -18 \\ 1 & 8 & 1 & 10 \end{bmatrix} R_1 \Leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 8 & 1 & 10 \\ 0 & -19 & 6 & -18 \end{bmatrix} \xrightarrow{R_2 \cdot (-1/19)} \begin{bmatrix} 1 & 8 & 1 & 10 \\ 0 & 1 & -6/19 & 18/19 \end{bmatrix} \xrightarrow{R_1 - 8R_2} \begin{bmatrix} 1 & 0 & 67/19 & 46/19 \\ 0 & 1 & -6/19 & 18/19 \end{bmatrix}$$

LET $z = t \Rightarrow y = \frac{18}{19} + \frac{6}{19}t$, $x = \frac{46}{19} - \frac{67}{19}t$

∴ LINE OF INTERSECTION

$(x, y, z) = \left(\frac{46}{19} - \frac{67}{19}t, \frac{18}{19} + \frac{6}{19}t, t\right)$ $t \in \mathbb{R}$.

Question 2. *(2 marks)* Find the equations of the line that passes through the point $P(3, -4, 0)$ and is parallel to both planes in question 1.

FROM 1) $\vec{v} = \left(-\frac{67}{19}, \frac{6}{19}, 1\right)$

∴ $(x, y, z) = \left(3 - \frac{67}{19}t, -4 + \frac{6}{19}t, t\right)$ $t \in \mathbb{R}$

Question 3. *(4 marks)* Find the equation of the plane containing the lines $(x, y, z) = (3 + 7t, 4 + 2t, 1 - t)$ and $(x, y, z) = (7t, -2 + 2t, 5 - t)$

$P_1(3, 4, 1)$ IS ON THE FIRST LINE, $P_2(0, -2, 5)$ IS ON THE SECOND LINE ∴ $\overrightarrow{P_1P_2} = (-3, -6, 4)$

DIRECTION VECTOR OF LINE $\vec{v} = (7, 2, -1)$

∴ $\vec{n} = \overrightarrow{P_1P_2} \times \vec{v} = \left(\begin{vmatrix} -6 & 2 \\ 4 & -1 \end{vmatrix}, -\begin{vmatrix} -3 & 7 \\ 4 & -1 \end{vmatrix}, \begin{vmatrix} -3 & 7 \\ -6 & 2 \end{vmatrix}\right) = (-2, 25, 36)$

∴ EQUATION OF PLANE: $-2x + 25y + 36z + d = 0$

$0 + 25(-2) + 36(5) + d = 0 \Rightarrow d = -130$

∴ $-2x + 25y + 36z - 130 = 0$