Dawson College: **Linear Algebra**: 201-NYC-05 06

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**Last Name:** SOLUTIONS

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# Quiz 10 (B)

**Question 1.** *(4 marks)* Do the planes $3x+5y+9z-12=0$ and $2x-3y+8z-2=0$ intersect? If so find the intersection.

$\vec{n_1} = (3,5,9)$ , $\vec{n_2} = (2,-3,8)$ ∴ THE PLANES ARE NOT PARALLEL

∴ THE PLANES INTERSECT.

$$\begin{bmatrix} 3 & 5 & 9 & 12 \\ 2 & -3 & 8 & 2 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 8 & 1 & 10 \\ 2 & -3 & 8 & 2 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 8 & 1 & 10 \\ 0 & -19 & 6 & -18 \end{bmatrix} R_2 \cdot (-1/19)$$

$$\begin{bmatrix} 1 & 8 & 1 & 10 \\ 0 & 1 & -6/19 & 18/19 \end{bmatrix} \xrightarrow{R_1 - 8R_2} \begin{bmatrix} 1 & 0 & 67/19 & 46/19 \\ 0 & 1 & -6/19 & 18/19 \end{bmatrix}$$

LET $z = t \Rightarrow y = \frac{18}{19} + \frac{6}{19}t$ , $x = \frac{46}{19} - \frac{67}{19}t$

∴ LINE OF INTERSECTION

$(x,y,z) = (46/19 - 67/19\,t,\ 18/19 + 6/19\,t,\ t)$ $t \in \mathbb{R}$

**Question 2.** *(2 marks)* Find the equations of the line that passes through the point $P(2,0,-12)$ and is parallel to both planes in question 1.

FROM 1) $\vec{v} = (-67/19,\ 6/19,\ 1)$

∴ LINE OF INTERSECTION:

$(x,y,z) = (2 - \frac{67}{19}t,\ 6/19\,t,\ -12 + t)$ $t \in \mathbb{R}$

**Question 3.** *(4 marks)* Find the equation of the plane containing the lines $(x,y,z) = (1+5t, 2-t, -2-4t)$ and $(x,y,z) = (5t, -1-t, 7-4t)$

$P_1(1,2,-2)$ IS ON THE FIRST LINE, $P_2(0,-1,7)$ IS ON THE SECOND LINE ∴ $\overrightarrow{P_1P_2} = (-1,-3,9)$

DIRECT VECTOR OF LINE $\vec{v} = (5,-1,-4)$

$\therefore \vec{n} = \overrightarrow{P_1P_2} \times \vec{v} = \left(\begin{vmatrix} -3 & -1 \\ 9 & -4 \end{vmatrix}, -\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix}, \begin{vmatrix} -1 & 5 \\ -3 & -1 \end{vmatrix}\right) = (21, 41, 16)$

∴ EQUATION OF LINE $21x + 41y + 16z + d = 0$

$21(0) + 41(-1) + 16(7) + d = 0 \Rightarrow d = -71$

$\therefore 21x + 41y + 16z - 71 = 0$