Dawson College: Linear Algebra: 201-NYC-05 06

April 29, 2010

Last Name: SOLUTIONS

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Quiz 11 (A)

Question 1. *(4 marks)* Let V be the set of pairs of integers (x,y). Define addition to be $(x_1,y_1)+(x_2,y_2)=(x_1x_2,\ y_1)$ and scalar multiplication to be $k(x,y)=(kx,ky)$. Is V a vector space?

NO. FAILS AXIOM 2.

LET $u=(x_1,y_1)$, $v=(x_2,y_2)$ BE ELEMENTS OF V.

THEN $u+v=(x_1,y_1)+(x_2,y_2)=(x_1x_2,y_1)$

$v+u=(x_2,y_2)+(x_1,y_1)=(x_2x_1,y_2)$

$\therefore\ u+v\neq v+u$ (TAKE $u=(0,1)$, $v=(1,0)$)

$\therefore$ NOT A VECTOR SPACE

Question 2. *(6 marks)* Check axiom 1 (closure under addition) and axiom 6 (closure under scalar multiplication) for the following:

(a) The set of 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a+d=b+c$ with the usual matrix addition and scalar multiplication.

(b) The set of triples of the form $(x,0,-1)$ with the usual addition and scalar multiplication.

a) LET $u=\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, $a_1+d_1=b_1+c_1$ $\quad v=\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$, $a_2+d_2=b_2+c_2$

- THE $u+v=\begin{bmatrix} a_1+a_2 & b_1+d_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$ AND $(a_1+a_2)+(d_1+d_2)$

$=(a_1+d_1)+(a_2+d_2)$

$=(b_1+c_1)+(b_2+c_2)$

$=(b_1+b_2)+(c_1+c_2)$ AS REQUIRED

$\therefore$ $u+v$ IS IN THE SET

$\therefore$ CLOSED UNDER ADDITION

- $ku=\begin{bmatrix} ka_1 & kb_1 \\ kc_1 & kd_1 \end{bmatrix}$ $\quad ka_1+kd_1=k(a_1+d_1)=k(b_1+c_1)$

$=kb_1+kc_1$ AS REQUIRED

$\therefore$ ku IS IN THE SET $\quad\therefore$ CLOSED UNDER SCALAR

5) LET $u = (x_1, 0, -1)$ $\quad v = (x_2, 0, -1)$

THEN $u + v = (x_1 + x_2, 0, -2)$ WHICH IS NOT IN THE SET

$\therefore$ NOT CLOSED UNDER ADDITION

• $ku = (kx_1, 0, k(-1))$ WHICH IS NOT IN THE SET

(IF $k \neq 1$) $\therefore$ NOT CLOSED UNDER SCALAR MULTIPLICATION.