Dawson College: Linear Algebra: 201-NYC-05 06 April 29, 2010

Last Name: SOLUTIONS

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Quiz 11 (B)

Question 1. *(4 marks)* Let V be the set of pairs of integers (x,y). Define addition to be $(x_1,y_1)+(x_2,y_2)=(x_1,\ y_1y_2)$ and scalar multiplication to be $k(x,y)=(kx,ky)$. Is V a vector space?

NO. FAILS AXIOM 2

LET $u=(x_1,y_1)$, $v=(x_2,y_2)$ BE ELEMENTS OF V

THEN $u+v=(x_1,y_1)+(x_2,y_2)=(x_1,\ y_1y_2)$

$v+u=(x_2,y_2)+(x_1,y_1)=(x_2,\ y_2y_1)$

$\therefore u+v\neq v+u$ (TAKE $u=(0,1)$, $v=(1,0)$)

$\therefore$ NOT A VECTOR SPACE.

Question 2. *(6 marks)* Check axiom 1 (closure under addition) and axiom 6 (closure under scalar multiplication) for the following:

(a) The set of triples of the form $(-1,0,x)$ with the usual addition and scalar multiplication.

(b) The set of 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a+d=b+c$ with the usual matrix addition and scalar multiplication.

a) LET $u=(-1,0,x_1)$, $v=(-1,0,x_2)$ THEN

- $u+v=(-2,0,x_1+x_2)$ WHICH IS NOT IN THE SET.
  $\therefore$ NOT CLOSED UNDER ADDITION
- $ku=(-k,0,kx_1)$ WHICH IS NOT IN THE SET (IF $k\neq 1$)
  $\therefore$ NOT CLOSED UNDER SCALAR MULTIPLICATION.

b) LET $u=\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ $a_1+d_1=b_1+c_1$, $v=\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ $a_2+d_2=b_2+c_2$

- $u+v=\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$ $(a_1+a_2)+(d_1+d_2)=(a_1+d_1)+(a_2+d_2)$
  $=(b_1+c_1)+(b_2+c_2)=(b_1+b_2)+(c_1+c_2)$
  AS REQUIRED

$\therefore u+v$ IS IN THE SET $\therefore$ CLOSED UNDER ADDITION

- $ku = \begin{bmatrix} ka_1 & kb_1 \\ kc_1 & kd_1 \end{bmatrix}$ $\quad ka_1 + kd_1 = k(a_1+d_1) = k(b_1+c_1)$

$= kb_1 + kc_1$ AS REQUIRED

$\therefore$ KU IS IN THE SET

$\therefore$ CLOSED UNDER SCALAR MULTIPLICATION.