

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 2 (A)

Question 1. (8 marks) Solve the following system of equations using Gauss-Jordan elimination:

$$\begin{aligned} 2x_1 + 4x_2 + 2x_4 &= 6 \\ x_1 + 2x_2 - x_3 &= 1 \\ -2x_1 - 4x_2 + 2x_3 &= -2 \end{aligned}$$

AUGMENTED MATRIX:

$$\begin{bmatrix} 2 & 4 & 0 & 2 & 6 \\ 1 & 2 & -1 & 0 & 1 \\ -2 & -4 & 2 & 0 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 2 & 4 & 0 & 2 & 6 \\ -2 & -4 & 2 & 0 & -2 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 + 2R_1}} \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \cdot (1/2)}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 2 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

REDUCED ROW ECHELON FORM

FREE VARIABLES:

LET $x_2 = s$, $x_4 = t$

$$\begin{aligned} x_3 + x_4 &= 2 \\ x_3 + t &= 2 \\ x_3 &= 2 - t \end{aligned}$$

$$\begin{aligned} x_1 + 2x_2 + x_4 &= 3 \\ x_1 + 2s + t &= 3 \\ x_1 &= 3 - 2s - t \end{aligned}$$

SOLUTION SET:

$$\begin{aligned} (x_1, x_2, x_3, x_4) &= \\ (3 - 2s - t, s, 2 - t, t) \end{aligned}$$

Question 2. (2 marks) Given:

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 \\ 4 & 1 \\ 7 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 4 \end{bmatrix}$$

Compute $3B^T - AC$ if possible.

$$\begin{aligned} 3B^T - AC &= 3 \begin{bmatrix} 1 & -2 \\ 4 & 1 \\ 7 & 0 \end{bmatrix}^T - \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 4 \end{bmatrix} = 3 \begin{bmatrix} 1 & 4 & 7 \\ -2 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 10 \\ 0 & 1 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 12 & 21 \\ -6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 10 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 14 & 11 \\ -6 & 2 & 3 \end{bmatrix} \end{aligned}$$