

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 2 (B)

Question 1. (8 marks) Solve the following system of equations using Gauss-Jordan elimination:

$$\begin{aligned} -2x_1 - 4x_2 + 2x_3 &= -2 \\ 2x_1 + 4x_2 + 2x_4 &= 6 \\ x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

AUGMENTED MATRIX:

$$\begin{bmatrix} -2 & -4 & 2 & 0 & -2 \\ 2 & 4 & 0 & 2 & 6 \\ 1 & 2 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 2 & 4 & 0 & 2 & 6 \\ -2 & -4 & 2 & 0 & -2 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 + 2R_1}} \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{2 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 2 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

FREE VARIABLES:

LET $x_2 = s$, $x_4 = t$

REDUCED ROW
ECHOLON FORM \therefore SOLUTION SET

$$\begin{aligned} \bullet x_3 + x_4 &= 2 \\ x_3 + t &= 2 \\ x_3 &= 2 - t \end{aligned}$$

$$\begin{aligned} \bullet x_1 + 2x_2 + x_3 &= 3 \\ x_1 + 2s + t &= 3 \\ x_1 &= 3 - 2s - t \end{aligned}$$

$$\begin{aligned} (x_1, x_2, x_3, x_4) \\ = (3 - 2s - t, s, 2 - t, t) \end{aligned}$$

Question 2. (2 marks) Given:

$$A = \begin{bmatrix} 2 & -5 \\ 3 & 1 \\ 6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$$

Compute $CB - 2A^T$ if possible.

$$CB - 2A^T = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 5 \end{bmatrix} - 2 \begin{bmatrix} 2 & -5 \\ 3 & 1 \\ 6 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 3 & 1 & 13 \end{bmatrix} - 2 \begin{bmatrix} 2 & 3 & 6 \\ -5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 3 & 1 & 13 \end{bmatrix} - \begin{bmatrix} 4 & 6 & 12 \\ -10 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 & -7 \\ 13 & -1 & 13 \end{bmatrix}$$