

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 4 (B)

Question 1. (4 marks) Find A^{-1} if possible given:

$$A = \begin{bmatrix} 4 & 4 & 4 \\ 2 & -1 & -1 \\ 4 & 1 & 1 \end{bmatrix}, \quad [A|I] = \left[\begin{array}{ccc|ccc} 4 & 4 & 4 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 4 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \cdot 1/4}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/4 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 4 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - 4R_1}} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/4 & 0 & 0 \\ 0 & -3 & -3 & -1/2 & 1 & 0 \\ 0 & -3 & -3 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1/4 & 0 & 0 \\ 0 & -3 & -3 & -1/2 & 1 & 0 \\ 0 & 0 & 0 & -1/2 & -1 & 1 \end{array} \right] \leftarrow$$

A IS NOT INVERTABLE SINCE ITS RREF CANNOT BE I

Question 2. (3 marks) Find a matrix B such that

$$B^{-3} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 27 \end{bmatrix} \Rightarrow B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

Question 3. (3 marks) Find conditions that b_1 and b_2 must satisfy in order for the system to be consistent.

$$\begin{aligned} 6x_1 - 9x_2 &= b_1 \\ 2x_1 - 3x_2 &= b_2 \end{aligned}$$

AUGMENTED MATRIX:

$$\left[\begin{array}{cc|c} 6 & -9 & b_1 \\ 2 & -3 & b_2 \end{array} \right] \xrightarrow{R_1 \cdot 1/6} \left[\begin{array}{cc|c} 1 & -3/2 & b_1/6 \\ 2 & -3 & b_2 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & -3/2 & b_1/6 \\ 0 & 0 & b_2 - b_1/3 \end{array} \right]$$

$$\therefore b_2 - b_1/3 = 0 \quad \text{or} \quad 3b_2 = b_1$$